

# A New Measurement of $|V_{us}|$ from KTeV

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- Introduction to  $|V_{us}|$
- Measurements of  $K_L$  Branching Fractions
- Measurements of  $K_L$  Semileptonic Form Factors
- Results and conclusions

The KTeV Collaboration:

Arizona, Chicago, Colorado, Elmhurst, Fermilab, Osaka,  
Rice, UCLA, UCSD, Virginia, Wisconsin

# Unitarity Tests of CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \begin{array}{l} \text{Uncertainty on } \sum_j V_{ij}^2 \\ 0.2\% \\ 2.7\% \\ 30\% \end{array}$$

For first row, PDG quotes  $2.2 \sigma$  deviation from unitarity:

$$1 - \left( |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \right) = 0.0043 \pm 0.0019 \quad (\text{PDG 2002})$$

## PDG $|V_{ux}|$ Evaluations

$|V_{ud}| = 0.9734 \pm 0.0008$  from  $0^+ \rightarrow 0^+$  nuclear  $\beta$  decays, neutron decay

$|V_{ub}| = (3.6 \pm 0.7) \times 10^{-3}$  from semileptonic B decay

$|V_{us}| = 0.2196 \pm 0.0023$  from  $K^+$ ,  $K^0$  decays to  $\pi e \nu$  ( $\pi \mu \nu$  not used by PDG because of large uncertainties in form factor measurements).

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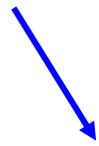
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Recent  $K^+$  measurement from BNL E865 consistent with unitarity.

 Interesting to revisit  $K_L$  measurements (PDG fit values based on averages of many old experiments with large errors)

# Determination of $|V_{us}|$ in Semileptonic $K_L$ Decays

KTeV measures  
 $B(K_L \rightarrow \pi e \nu)$  and  
 $B(K_L \rightarrow \pi \mu \nu)$



$\Gamma_{Kl3}$

$$= \frac{G_F^2 M_K^5}{192 \pi^3} \underbrace{S_{EW} (1 + \delta_K^l)}_{\text{Rad. Corrections (theory)}} |V_{us}|^2 f_+^2(0) I_K^l$$

Rad. Corrections  
 (theory)



KTeV measures  
 form factors needed  
 to calculate phase  
 space integrals



Form factor  
 at  $t=0$   
 (theory)



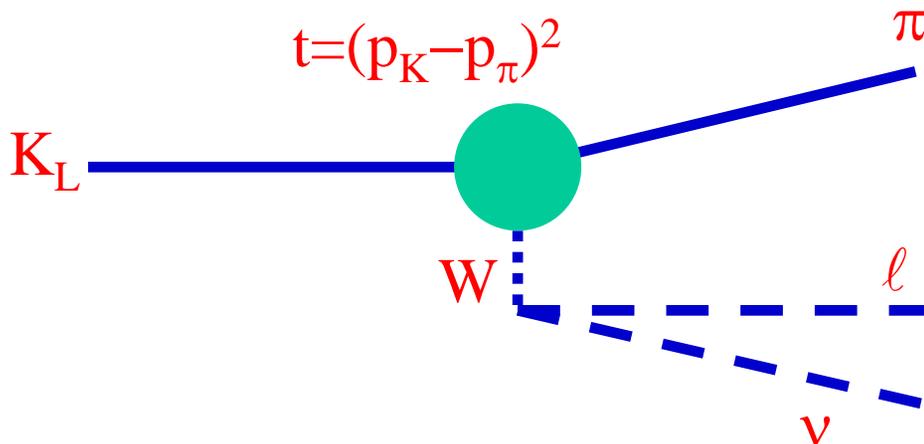
# KTeV Measurements

- To determine semileptonic partial widths, KTeV measures branching fractions for the 6 largest decay modes:

$$K_L \rightarrow \pi^\pm e^\mp \nu, K_L \rightarrow \pi^\pm \mu^\mp \nu, K_L \rightarrow \pi^+ \pi^- \pi^0, K_L \rightarrow \pi^0 \pi^0 \pi^0, \\ K_L \rightarrow \pi^+ \pi^-, K_L \rightarrow \pi^0 \pi^0$$

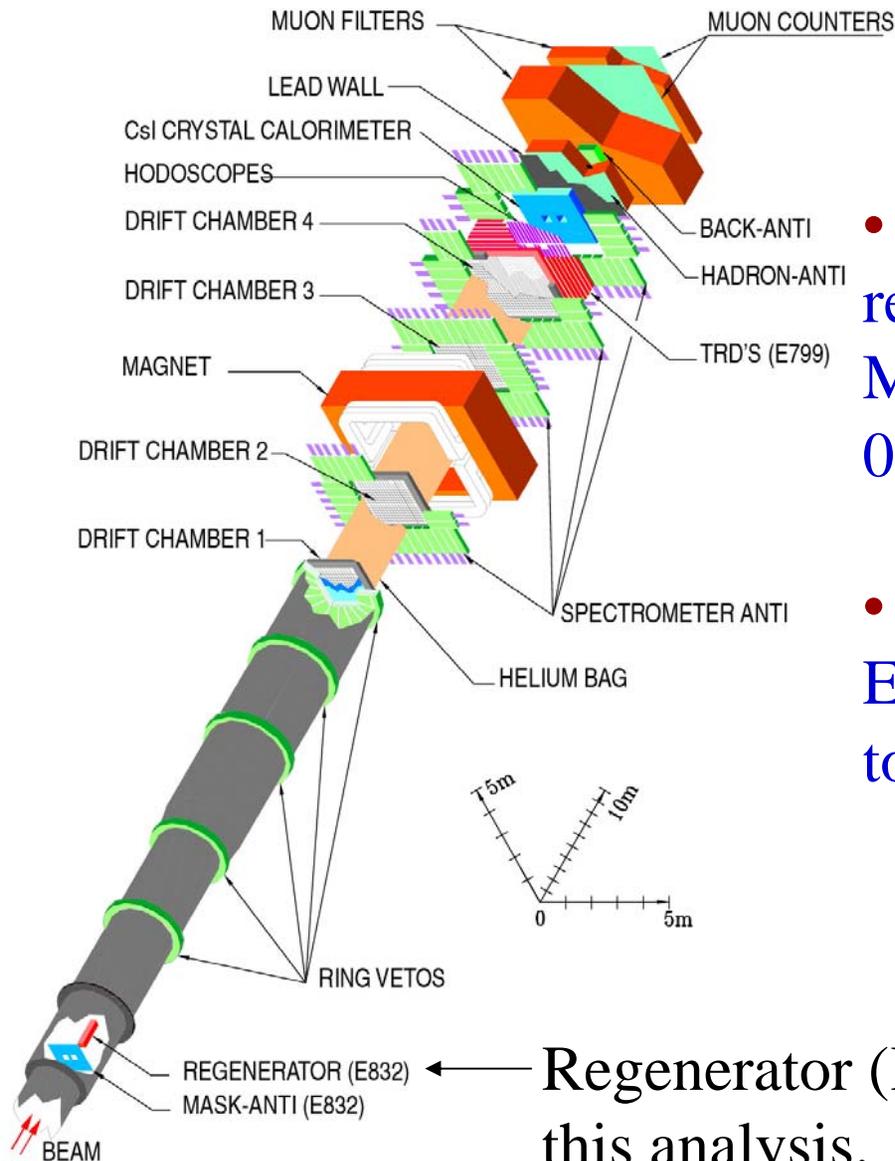
The  $K_L$  lifetime ( $\tau_L$ ) is used to convert branching fractions into partial widths

- Form factors describe the  $t$  distribution of decay:



We measure the two independent form factors for semileptonic decay, and determine the phase space integrals  $I_K$ .

# KTeV Detector



- Charged particle momentum resolution  $< 1\%$  for  $p > 8 \text{ GeV}/c$ ; Momentum scale known to  $0.01\%$  from  $K \rightarrow \pi^+ \pi^-$ .

- CsI energy resolution  $< 1\%$  for  $E_\gamma > 3 \text{ GeV}$ ; energy scale known to  $0.1\%$  from  $K \rightarrow \pi e \nu$ .

← Regenerator ( $K_S$ ) beam not used in this analysis.

## Material upstream of CsI Calorimeter

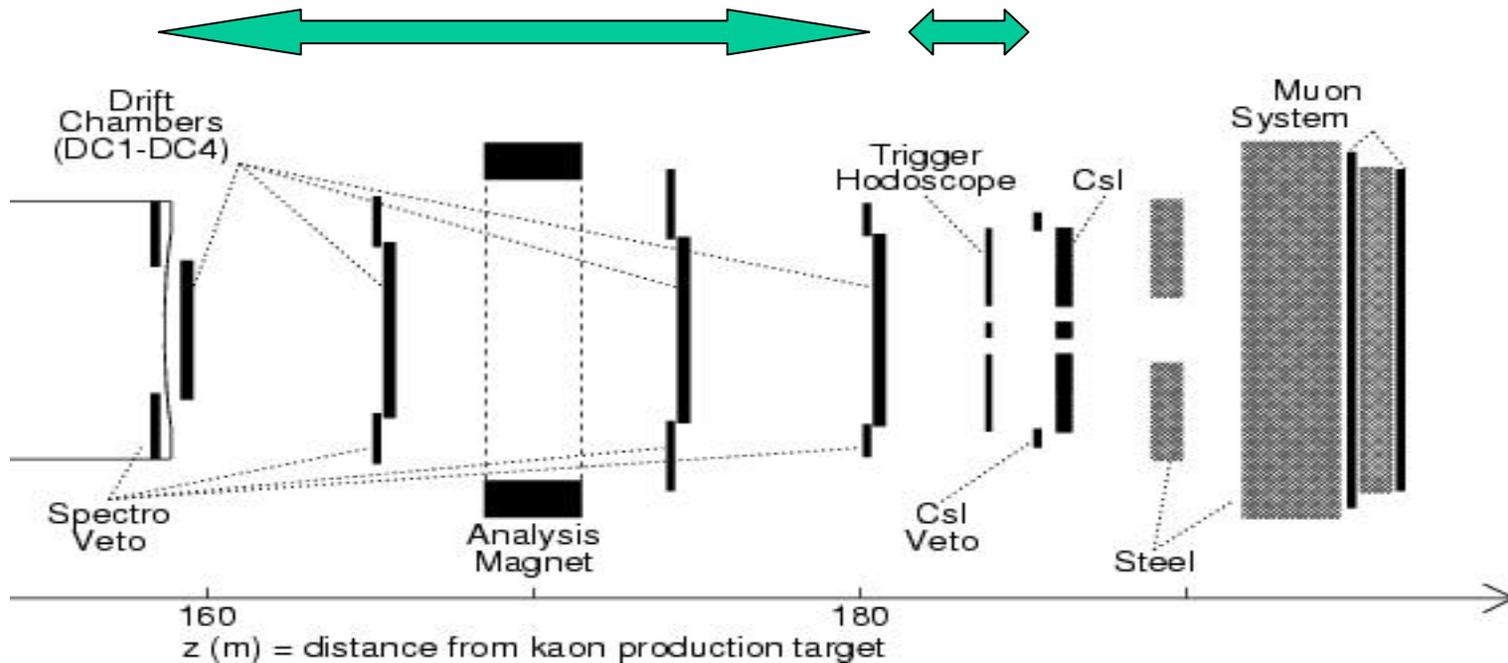
Spectrometer:

1.2% rad len

0.7%  $\pi$  int len

3.1% rad len

1.4%  $\pi$  int len



Material estimate checked with  $K \rightarrow \pi e \nu$  with external Brem.  
and  $K_L \rightarrow \pi^0 \pi^0 \pi^0$  with  $\gamma$  conversion.

To determine the semileptonic widths, we measure the following 5 ratios:

$$\Gamma_{K\mu 3} / \Gamma_{Ke 3} = \Gamma(K_L \rightarrow \pi^\pm \mu^\mp \nu) / \Gamma(K_L \rightarrow \pi^\pm e^\mp \nu)$$

$$\Gamma_{+-0} / \Gamma_{Ke 3} = \Gamma(K_L \rightarrow \pi^+ \pi^- \pi^0) / \Gamma(K_L \rightarrow \pi^\pm e^\mp \nu)$$

$$\Gamma_{000} / \Gamma_{Ke 3} = \Gamma(K_L \rightarrow \pi^0 \pi^0 \pi^0) / \Gamma(K_L \rightarrow \pi^\pm e^\mp \nu)$$

$$\Gamma_{+-} / \Gamma_{Ke 3} = \Gamma(K_L \rightarrow \pi^+ \pi^-) / \Gamma(K_L \rightarrow \pi^\pm e^\mp \nu)$$

$$\Gamma_{00} / \Gamma_{000} = \Gamma(K_L \rightarrow \pi^0 \pi^0) / \Gamma(K_L \rightarrow \pi^0 \pi^0 \pi^0)$$

These six decay modes account for 99.93% of  $K_L$  decays, so ratios may be combined to determine branching fractions.

E.g.,

$$B_{Ke 3} = \frac{0.9993}{1 + \frac{\Gamma_{K\mu 3}}{\Gamma_{Ke 3}} + \frac{\Gamma_{000}}{\Gamma_{Ke 3}} + \frac{\Gamma_{+-0}}{\Gamma_{Ke 3}} + \frac{\Gamma_{+-}}{\Gamma_{Ke 3}} + \frac{\Gamma_{00}}{\Gamma_{Ke 3}}}$$

## Features of Branching Fraction Analysis

- Each ratio measured in statistically independent data sample collected with a single trigger (samples sizes are  $10^5$  to  $10^6$  per decay mode)
- Each ratio measured in two data samples:
  - “high intensity” (same data used for  $\epsilon'/\epsilon$  analysis)
  - “low intensity” (no regenerator and  $\times 10$  lower intensity)Result for each ratio based on sample with lower total uncertainty
- Monte Carlo simulation is used to correct for acceptance difference between pair of modes
- Simulation includes inner bremsstrahlung contributions for all decay modes with charged particles, so branching fractions include radiated photons.

## Charged Decay Modes

$$K_L \rightarrow \pi^\pm e^\mp \nu, K_L \rightarrow \pi^\pm \mu^\mp \nu, K_L \rightarrow \pi^+ \pi^- \pi^0, K_L \rightarrow \pi^+ \pi^-$$

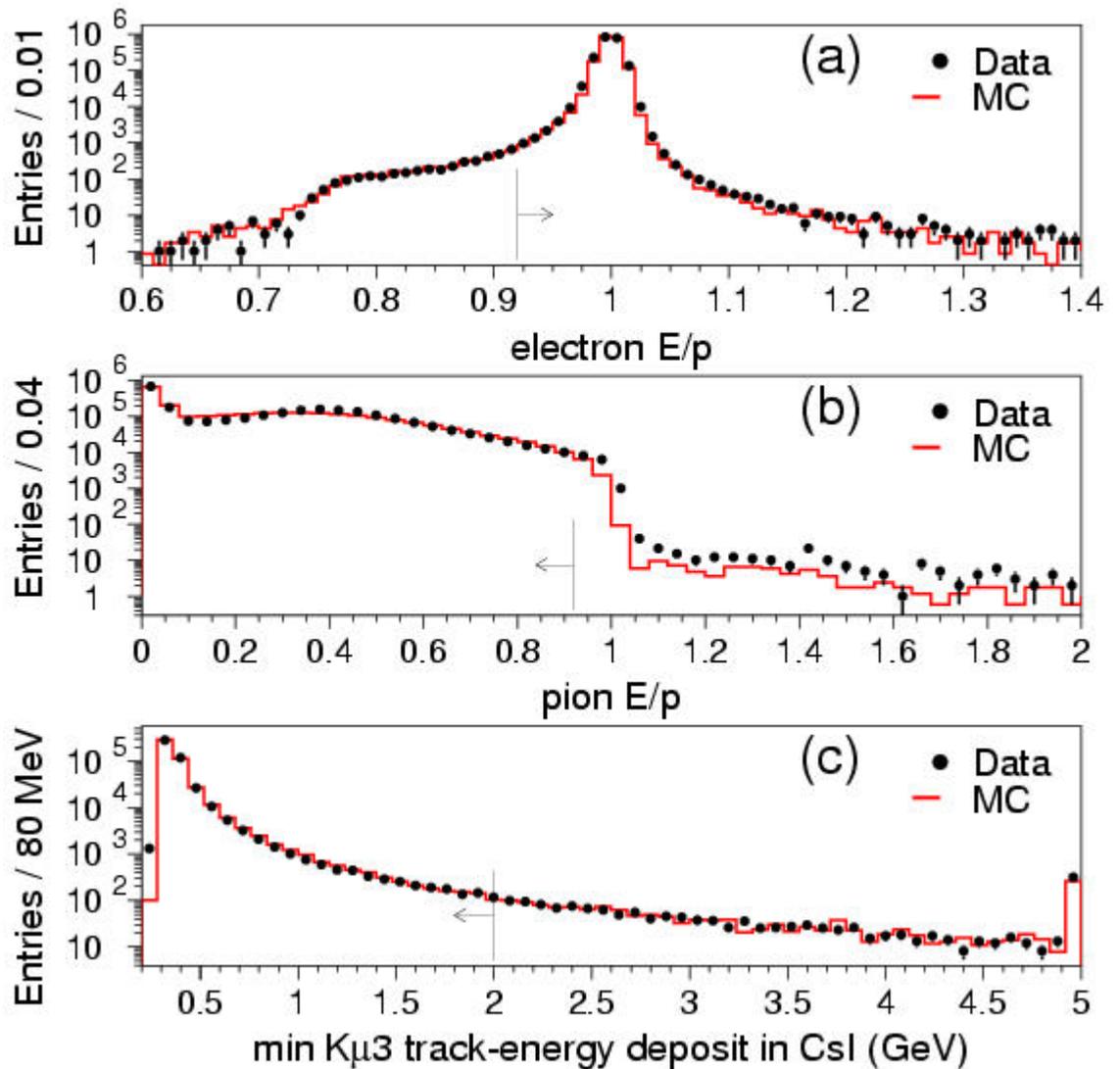
- Select two oppositely charged tracks from a single vertex.
- Use particle ID (calorimeter E and spectrometer p) and kinematics to separate 4 decay modes.

### Unusual features:

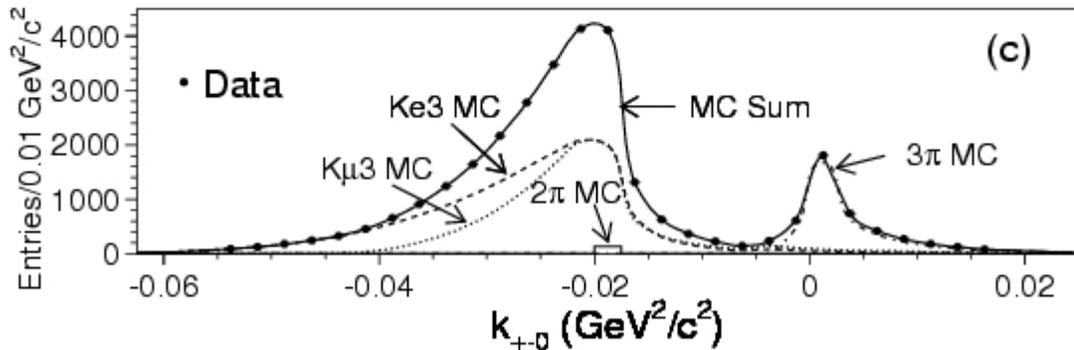
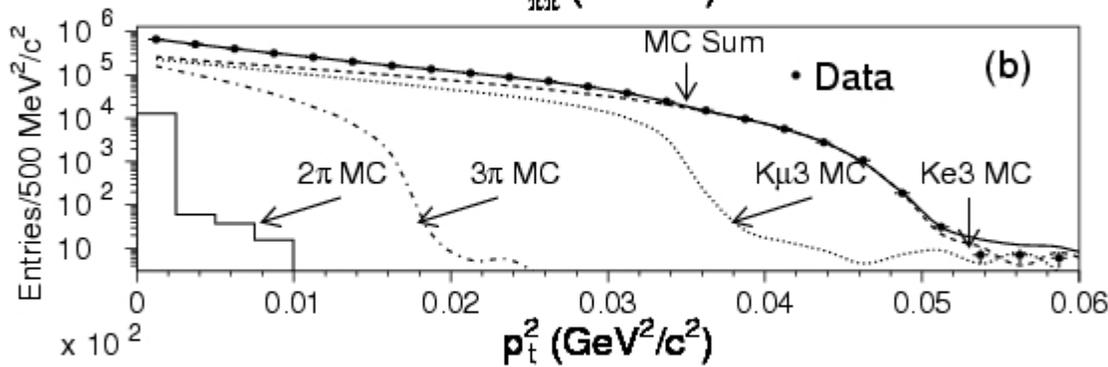
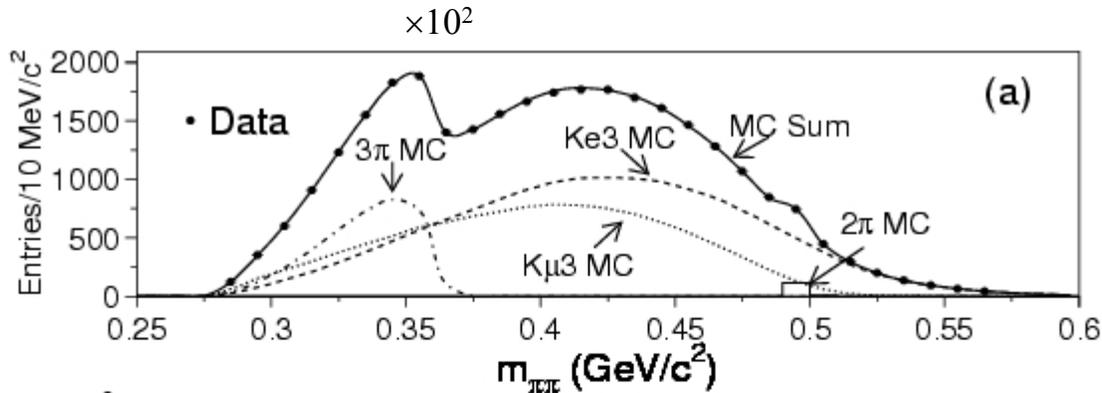
- For  $K_L \rightarrow \pi \mu \nu$ , we do not use muon system.
- For  $K_L \rightarrow \pi^+ \pi^- \pi^0$ , we do not reconstruct the  $\pi^0 \rightarrow \gamma \gamma$  decay.
- For partially reconstructed decays, there are multiple energy solutions; all solutions are required to be between 40 and 120 GeV.

# Particle Identification

Energy response tails measured in data and modeled in Monte Carlo.



# Kinematic Variables: $m_{\pi\pi}$ , $p_t^2$ , $k_{+-0}$

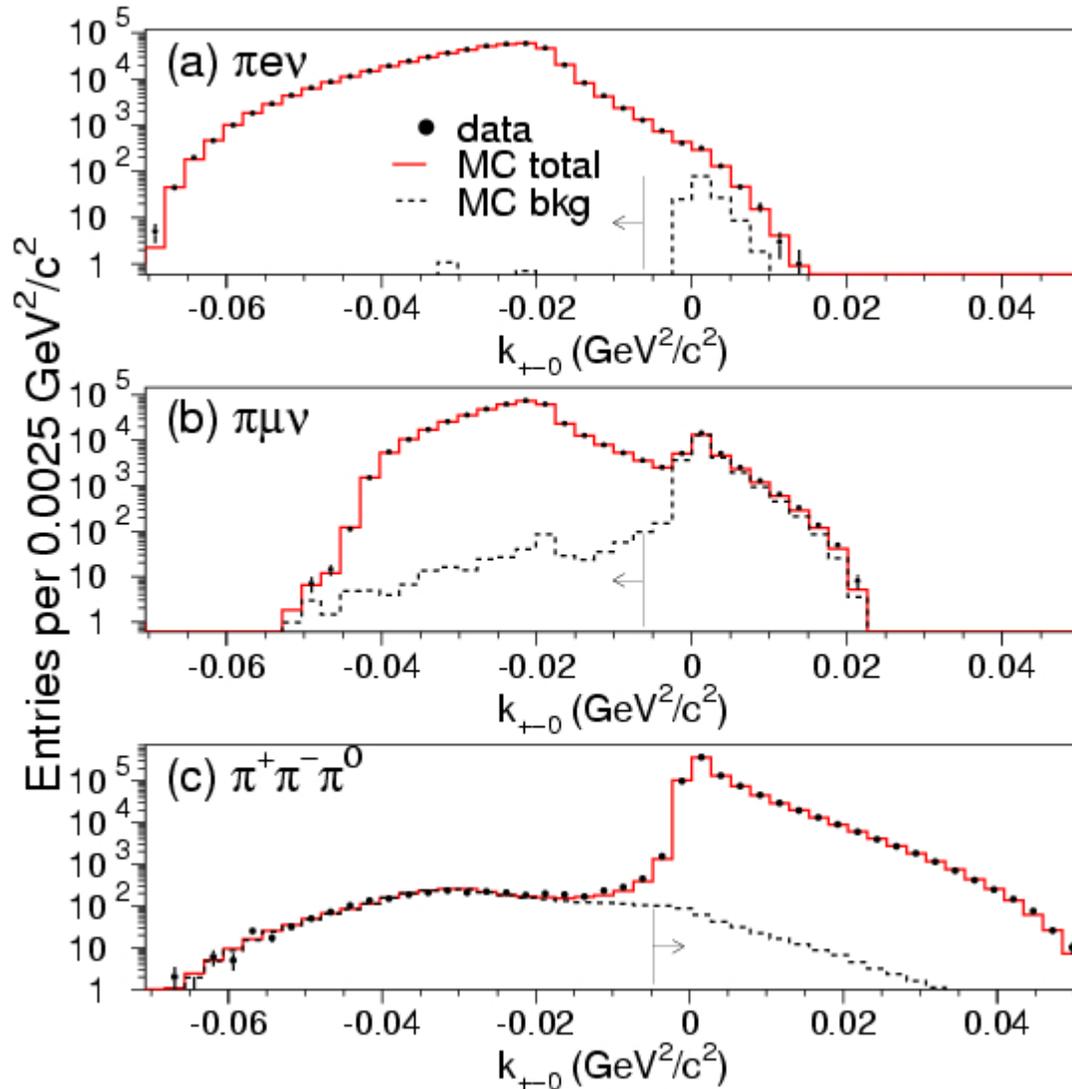


For  $K_L \rightarrow \pi^+ \pi^- \pi^0$  decays,  
 $k_{+-0}$  = longitudinal momentum squared of  $\pi^0$  in frame where  $\pi^+ \pi^-$  momentum is orthogonal to  $K$  momentum.

$$k_{+-0} > 0 \text{ for } K_L \rightarrow \pi^+ \pi^- \pi^0$$

$$k_{+-0} < 0 \text{ for } K_L \rightarrow \pi^\pm \ell^\mp \nu$$

# $k_{+-0}$ for Partially Reconstructed Decays

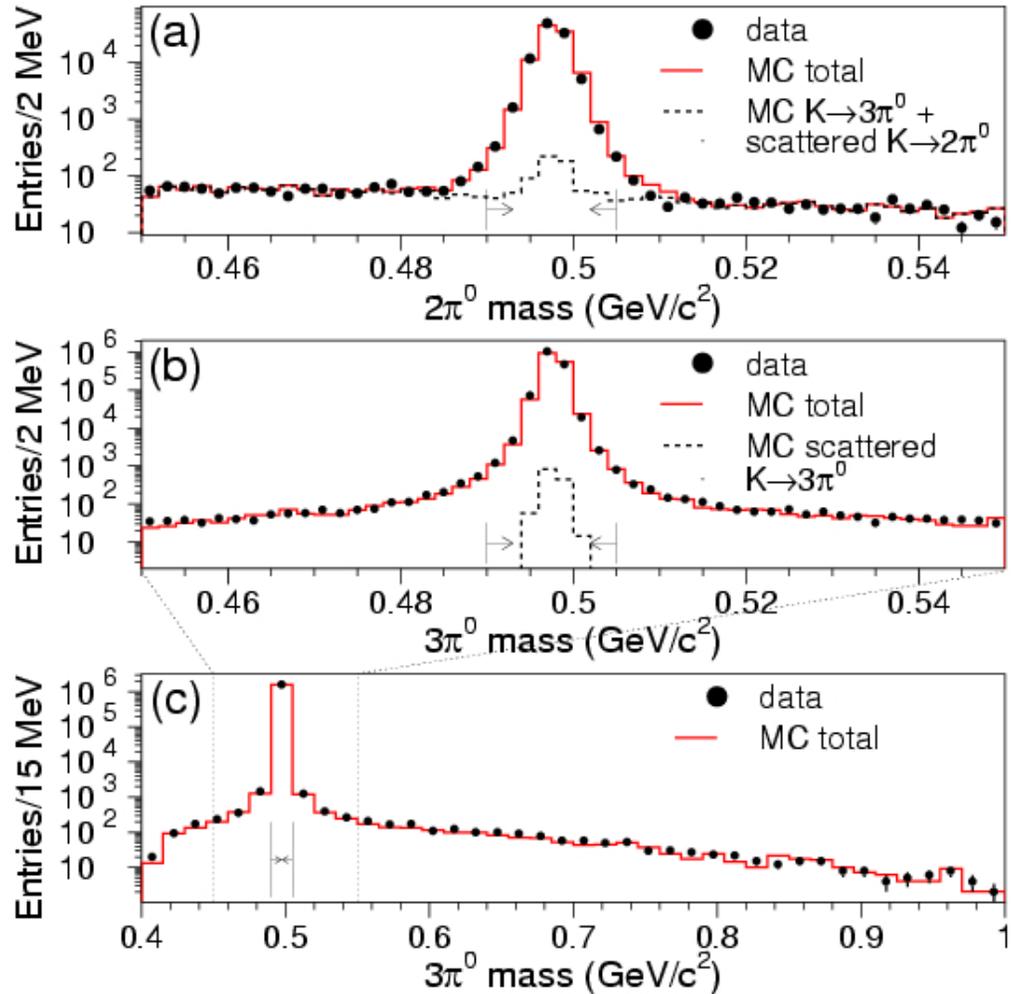


# Neutral Decay Modes

$$K_L \rightarrow \pi^0 \pi^0 \text{ and } K_L \rightarrow \pi^0 \pi^0 \pi^0$$

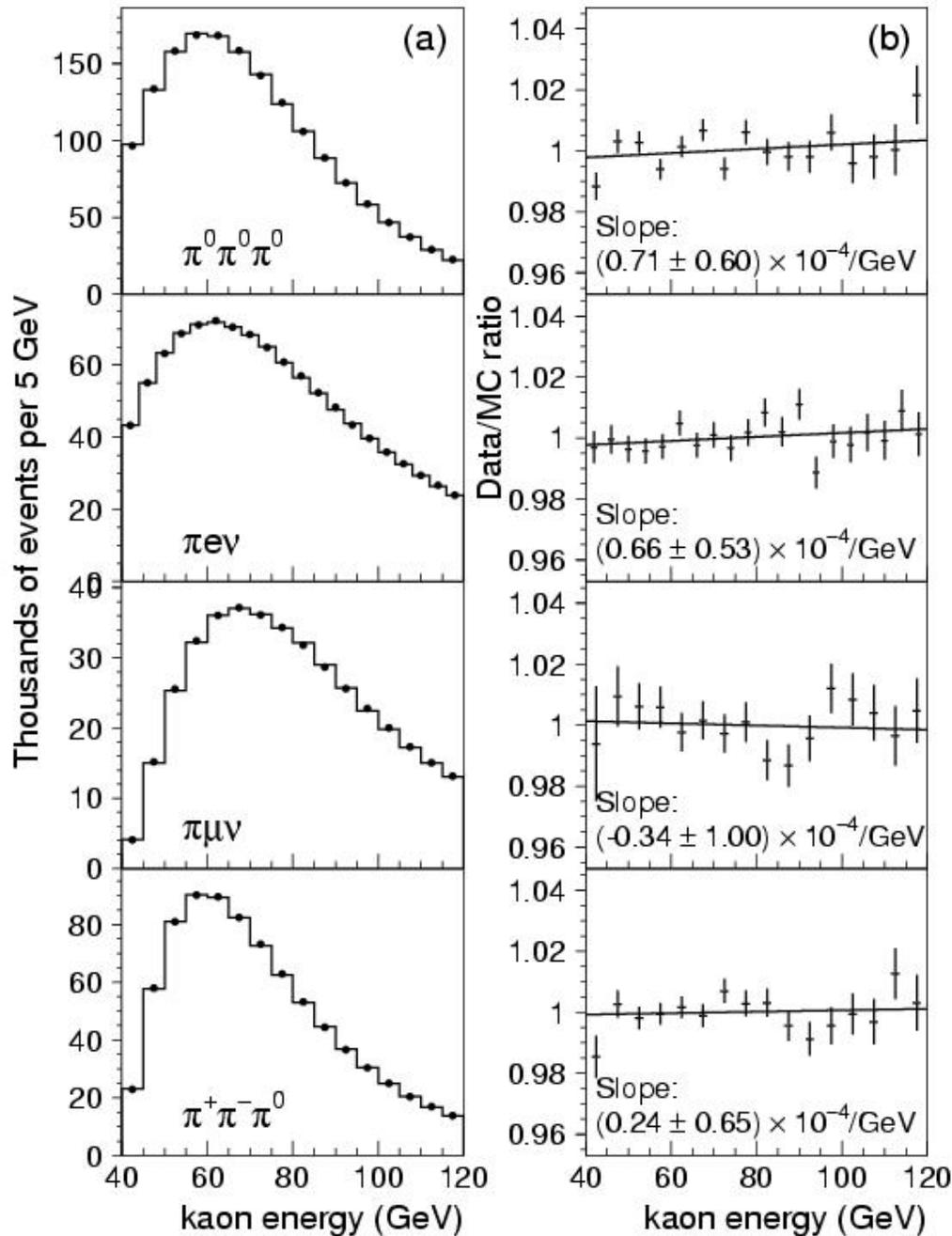
4 or 6 photon-like clusters are paired to reconstruct two or three neutral pions consistent with a single decay vertex.

(Analysis almost identical to  $\epsilon'/\epsilon$  analysis.)



## Backgrounds

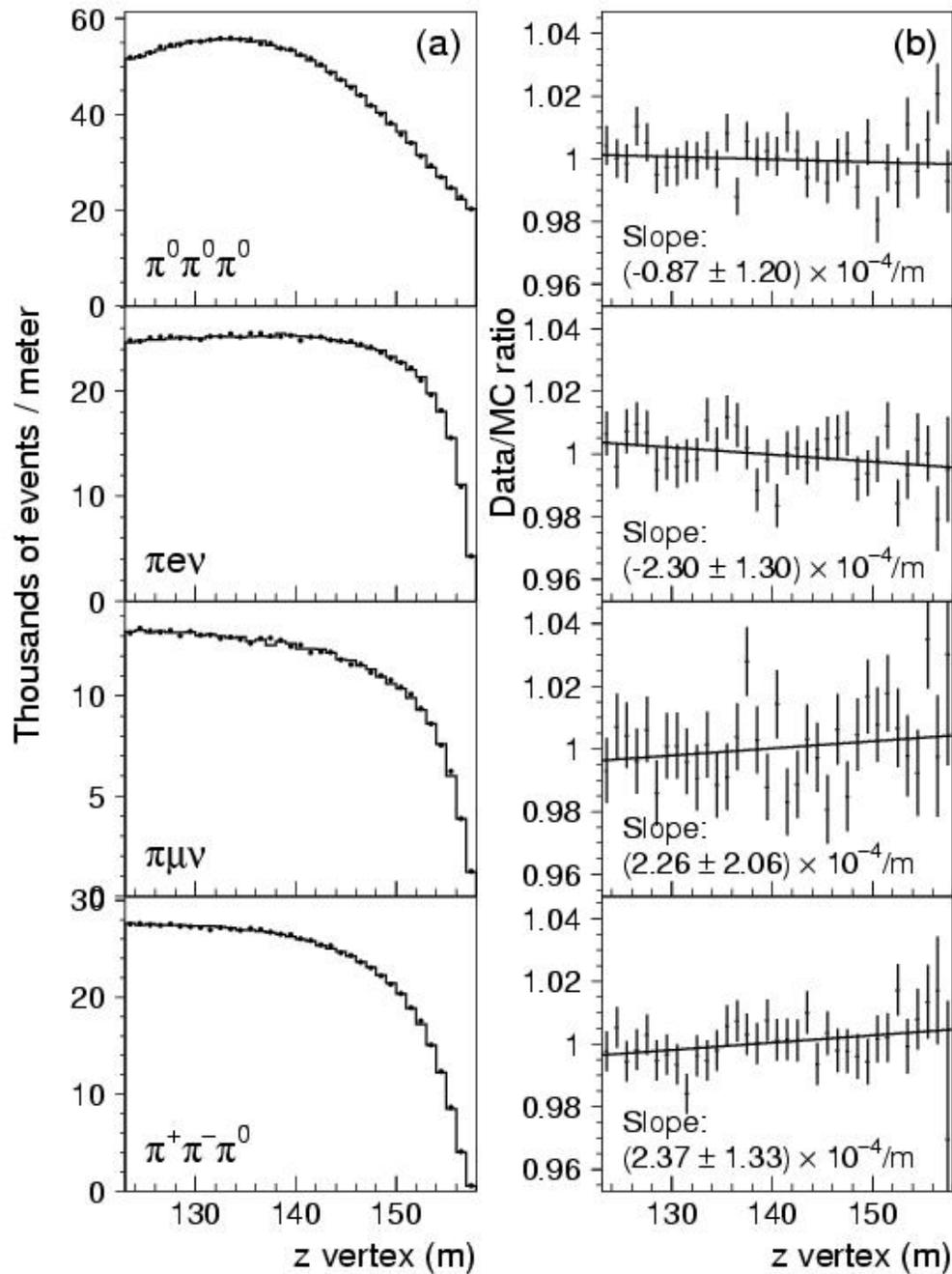
Decay Mode	Background (%)
$K_L \rightarrow \pi e \nu$	<b>&lt;0.003</b>
$K_L \rightarrow \pi \mu \nu$	<b>0.01</b>
$K_L \rightarrow \pi^+ \pi^- \pi^0$	<b>0.05</b>
$K_L \rightarrow \pi^0 \pi^0 \pi^0$	<b>None</b>
$K_L \rightarrow \pi^+ \pi^-$	<b>0.16</b>
$K_L \rightarrow \pi^0 \pi^0$	<b>0.71</b>



## Comparison of data and Monte Carlo kaon energy distributions

Monte Carlo spectrum was tuned using  $K_L \rightarrow \pi^+ \pi^-$  events

For partially reconstructed modes, high energy solution is plotted



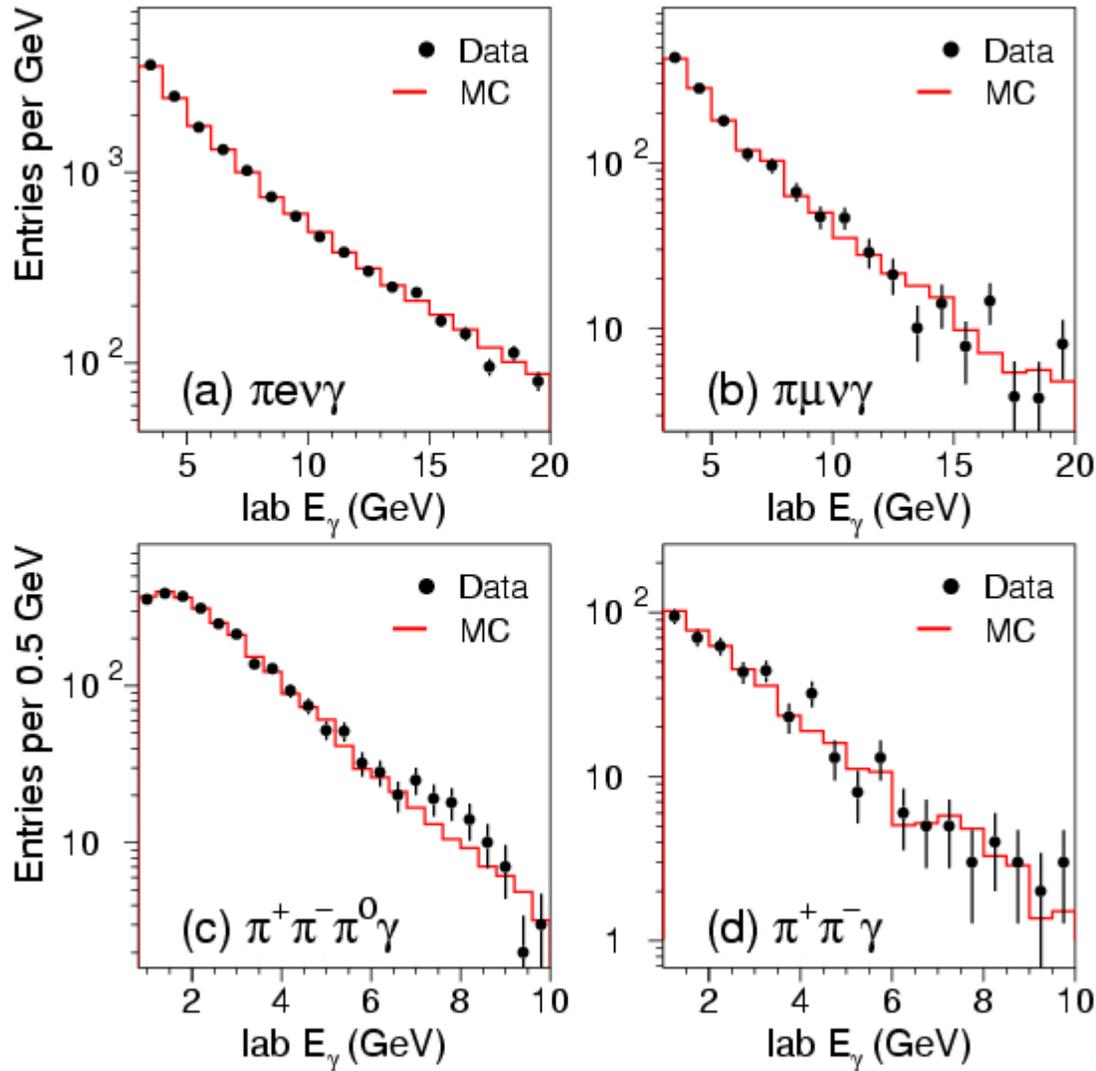
Comparison of data  
and Monte Carlo decay  
vertex distributions

## Branching fractions include radiated photons:

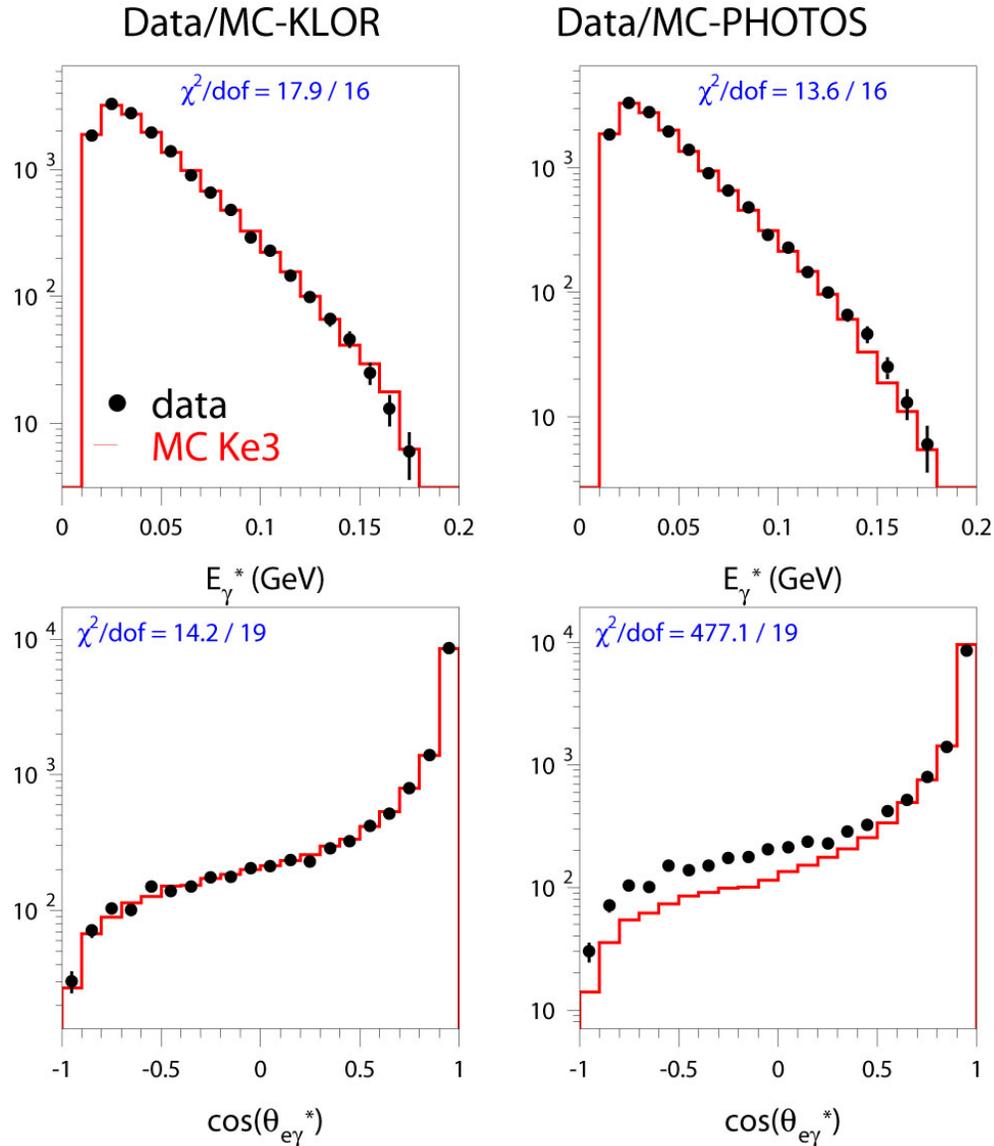
- $K_{e3(\gamma)}$  and  $K_{\mu3(\gamma)}$ : KLOR written by T. Andre. Includes virtual and real photons.
- $K_{+-0(\gamma)}$ : PHOTOS
- $K_{+-}(\gamma)$ : KTeV generator includes IB, but ignores direct emission.

Radiation changes  $Ke3$  acceptance by 3%; effect on other modes is  $< 0.5\%$ .

# Data – MC Comparison for Radiative Photon Candidates



# KLOR – PHOTOS Comparison for Semileptonic Decays



# Systematic Uncertainties in Partial Width Ratios in percent

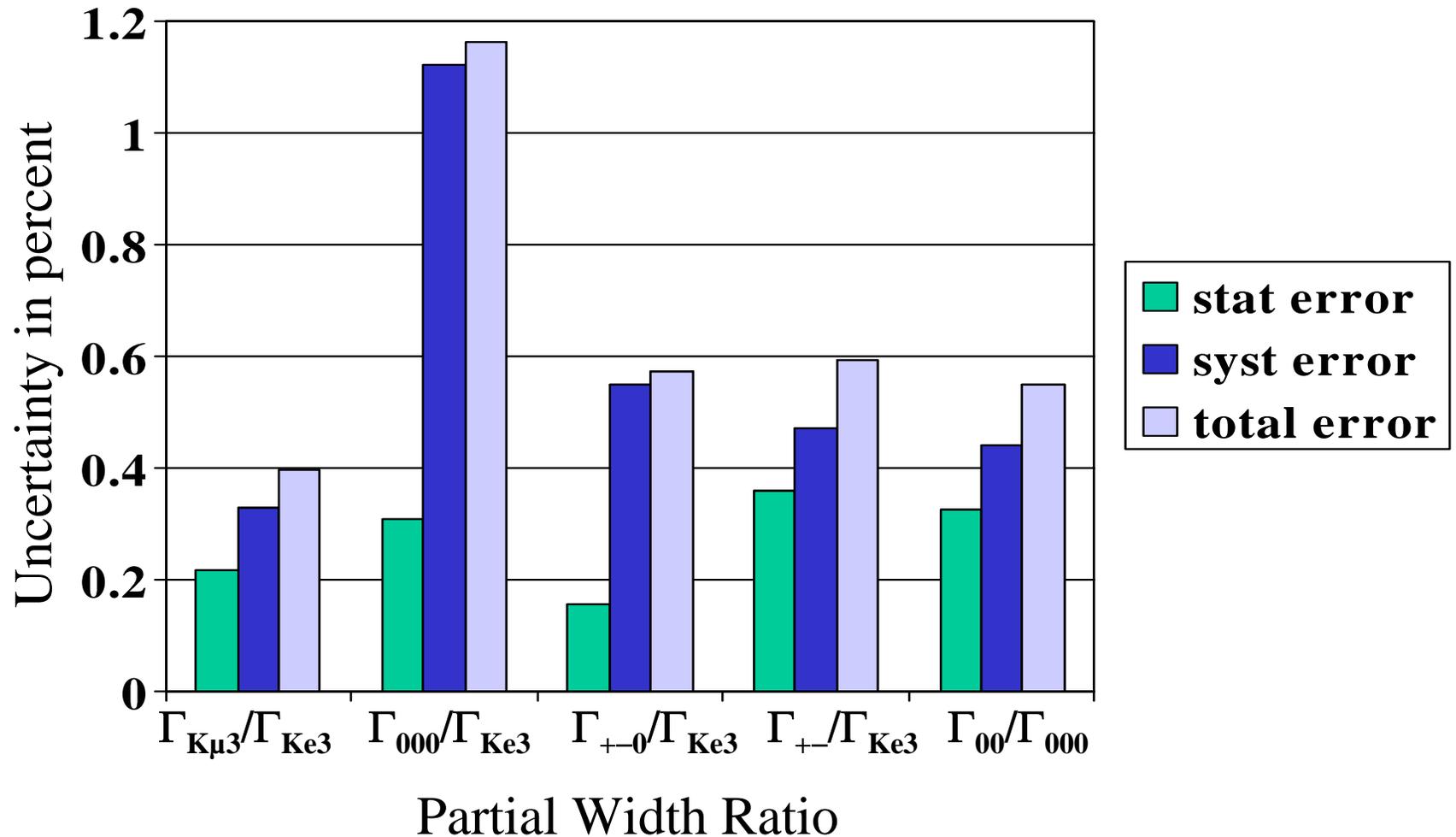
Source of uncertainty	$\Gamma_{K\mu 3} / \Gamma_{Ke 3}$	$\Gamma_{000} / \Gamma_{Ke 3}$	$\Gamma_{+-0} / \Gamma_{Ke 3}$	$\Gamma_{+-} / \Gamma_{Ke 3}$	$\Gamma_{00} / \Gamma_{000}$
Acceptance (MC Simulation)					
Event Generation:					
- Kaon energy spectrum	0.02	0.16	0.04	0.02	0.01
- Form factor	0.11	0.08	0.29	0.08	0.00
Radiative corrections:					
-	0.15	0.20	0.14	0.14	0.00
Particle Propagation:					
- Detector material	0.10	0.56	0.33	0.33	0.15
- Detector geometry	0.02	0.39	0.05	0.02	0.08
Detector Response:					
- Accidental activity	0.00	0.22	0.04	0.02	0.03
- Trigger	0.00	0.07	0.10	0.07	0.28
- $e^\pm, \mu^\pm, \pi^\pm$ reconstruction	0.21	0.70	0.24	0.26	0.00
- $\pi^0$ reconstruction	0.00	0.37	0.00	0.00	0.23
Background	0.10	0.00	0.02	0.04	0.04
$B(\pi^0 \rightarrow \gamma\gamma)$	0.00	0.10	0.10	0.00	0.03
Monte Carlo Statistics	0.10	0.12	0.05	0.13	0.16
Total	0.33	1.12	0.55	0.47	0.44

## Main Systematic Uncertainties

- $\Gamma_{000}/\Gamma_{Ke3}$  ratio:
  - $\Rightarrow$  0.6% two-track efficiency uncertainty
  - $\Rightarrow$  0.4% uncertainty in  $3\pi^0$  recon
  - $\Rightarrow$  0.6% uncertainty from detector material
- 0.35% uncertainty in loss from  $\pi$ -interactions for ratios with different number of charged  $\pi$

# Statistical and Systematic Errors in Partial Width Ratios

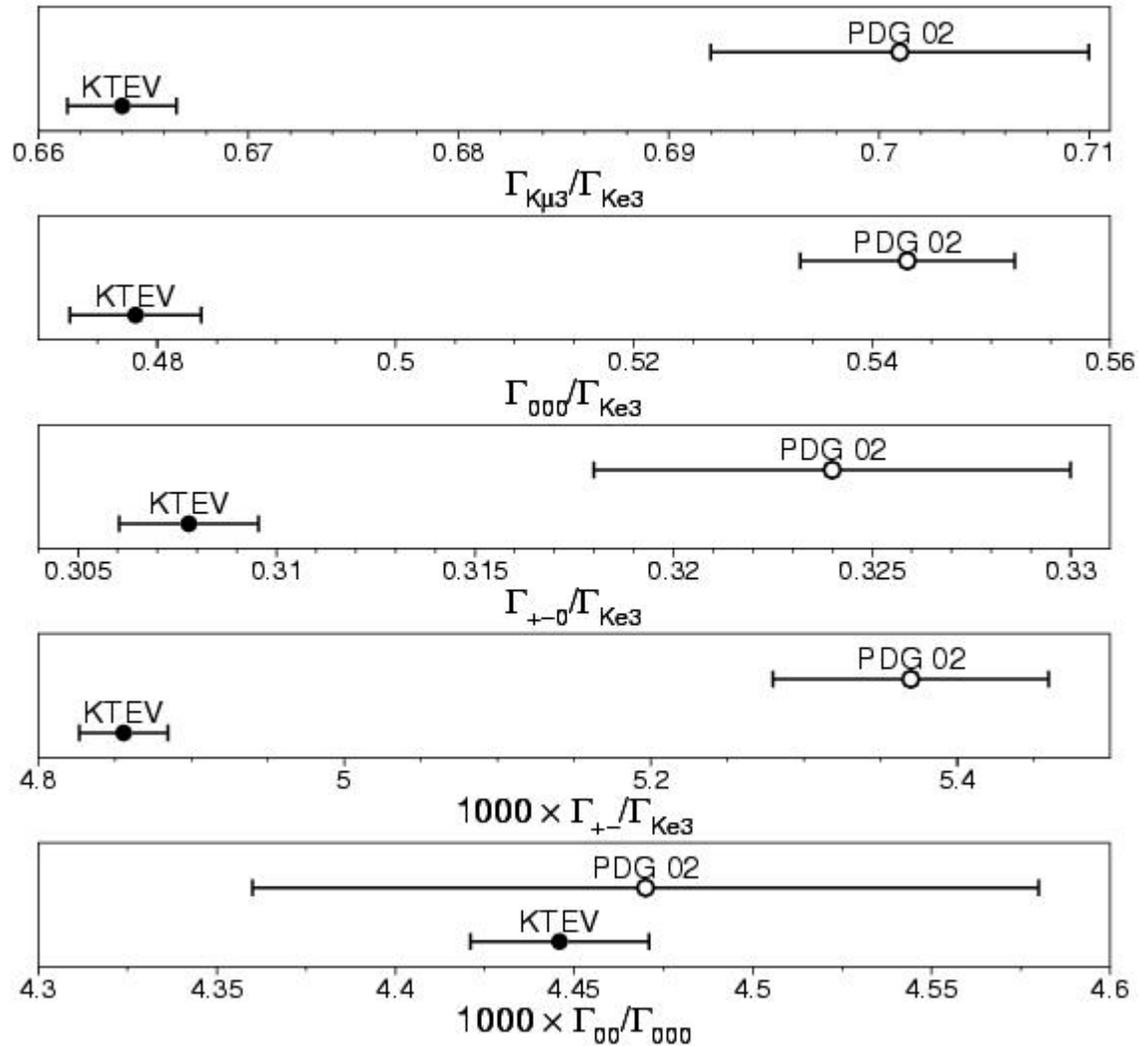
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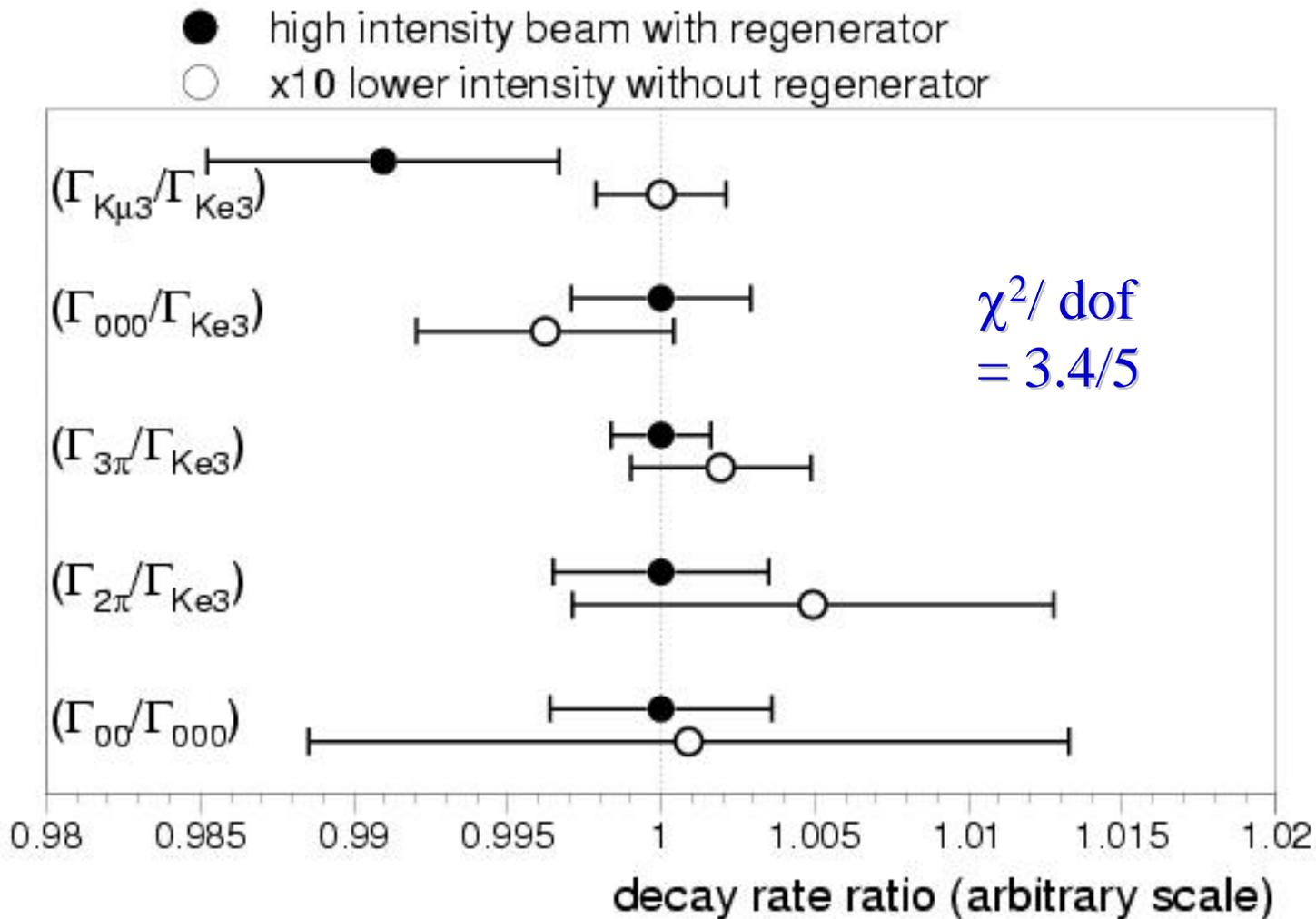
## Measured Partial Width Ratios

Decay Modes	Partial Width Ratio
$\Gamma_{K\mu 3} / \Gamma_{Ke 3}$	$0.6640 \pm 0.0014 \pm 0.0022$
$\Gamma_{000} / \Gamma_{Ke 3}$	$0.4782 \pm 0.0014 \pm 0.0053$
$\Gamma_{+-0} / \Gamma_{Ke 3}$	$0.3078 \pm 0.0005 \pm 0.0017$
$\Gamma_{+-} / \Gamma_{Ke 3}$	$(4.856 \pm 0.017 \pm 0.023) \times 10^{-3}$
$\Gamma_{00} / \Gamma_{000}$	$(4.446 \pm 0.016 \pm 0.019) \times 10^{-3}$

# Comparison of KTeV and PDG Partial Width Ratios



# Cross Checks: Width Ratios with High and Low Intensity



## Cross Checks: $K \rightarrow \pi \mu \nu$ and $K \rightarrow \pi^+ \pi^- \pi^0$ Analysis

- $K \rightarrow \pi \mu \nu$  yields without/with  $\mu$  system agree to  $(0.08 \pm 0.02_{\text{stat}})\%$
- $K \rightarrow \pi^+ \pi^- \pi^0$  yields without/with  $\pi^0 \rightarrow \gamma \gamma$  reconstruction in CsI agree to  $(0.03 \pm 0.028_{\text{stat}})\%$ .

**Acceptance in two analyses differs by factor of 5!**

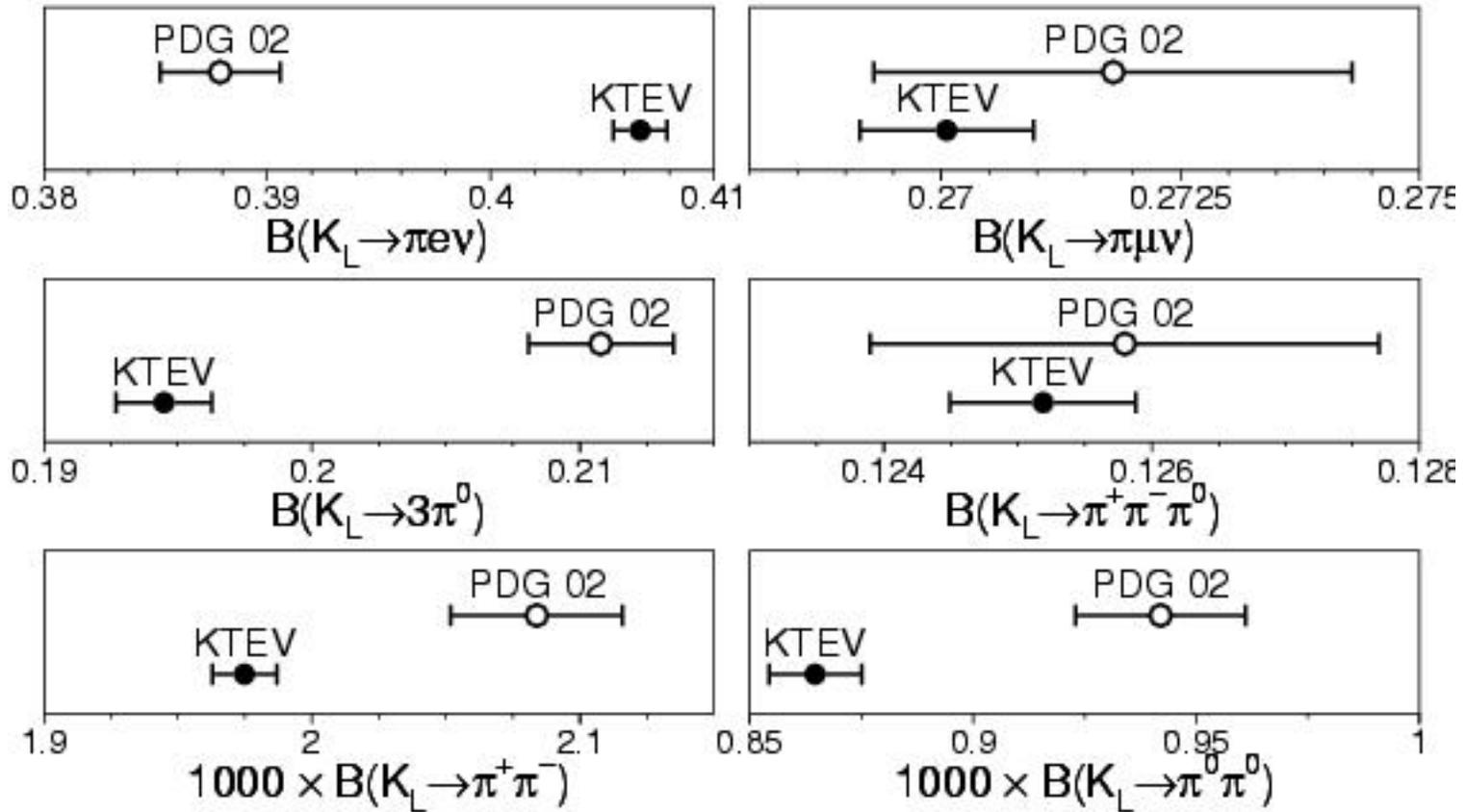
- $\Gamma_{+-0}/(\Gamma_{Ke3} + \Gamma_{K\mu3} + \Gamma_{+-0})$  may be determined by fitting the  $k_{+-0}$  distribution. The difference with the nominal analysis is  $(0.35 \pm 0.51)\%$ .

# Branching Fraction and Partial Width Results

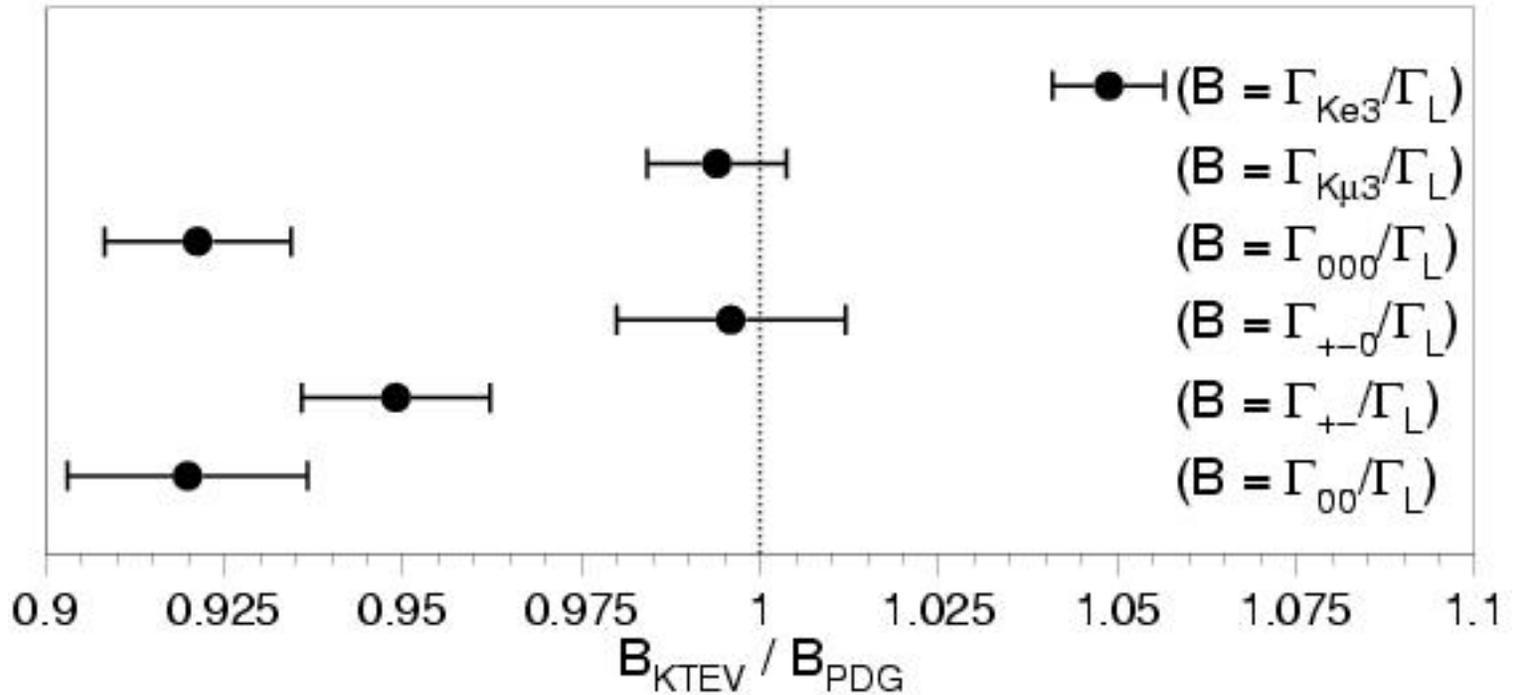
Decay Mode	Branching Fraction	$\Gamma_i (10^7 \text{ s}^{-1})$
$K_L \rightarrow \pi e \nu$	$0.4067 \pm 0.0011$	$0.7897 \pm 0.0065$
$K_L \rightarrow \pi \mu \nu$	$0.2701 \pm 0.0009$	$0.5244 \pm 0.0044$
$K_L \rightarrow \pi^+ \pi^- \pi^0$	$0.1252 \pm 0.0007$	$0.2431 \pm 0.0023$
$K_L \rightarrow \pi^0 \pi^0 \pi^0$	$0.1945 \pm 0.0018$	$0.3777 \pm 0.0045$
$K_L \rightarrow \pi^+ \pi^-$	$(1.975 \pm 0.012) \times 10^{-3}$	$(3.835 \pm 0.038) \times 10^{-3}$
$K_L \rightarrow \pi^0 \pi^0$	$(0.865 \pm 0.010) \times 10^{-3}$	$(1.679 \pm 0.024) \times 10^{-3}$

Partial widths use  $K_L$  lifetime of  $\tau_L = (5.15 \pm 0.04) \times 10^{-8}$  sec.

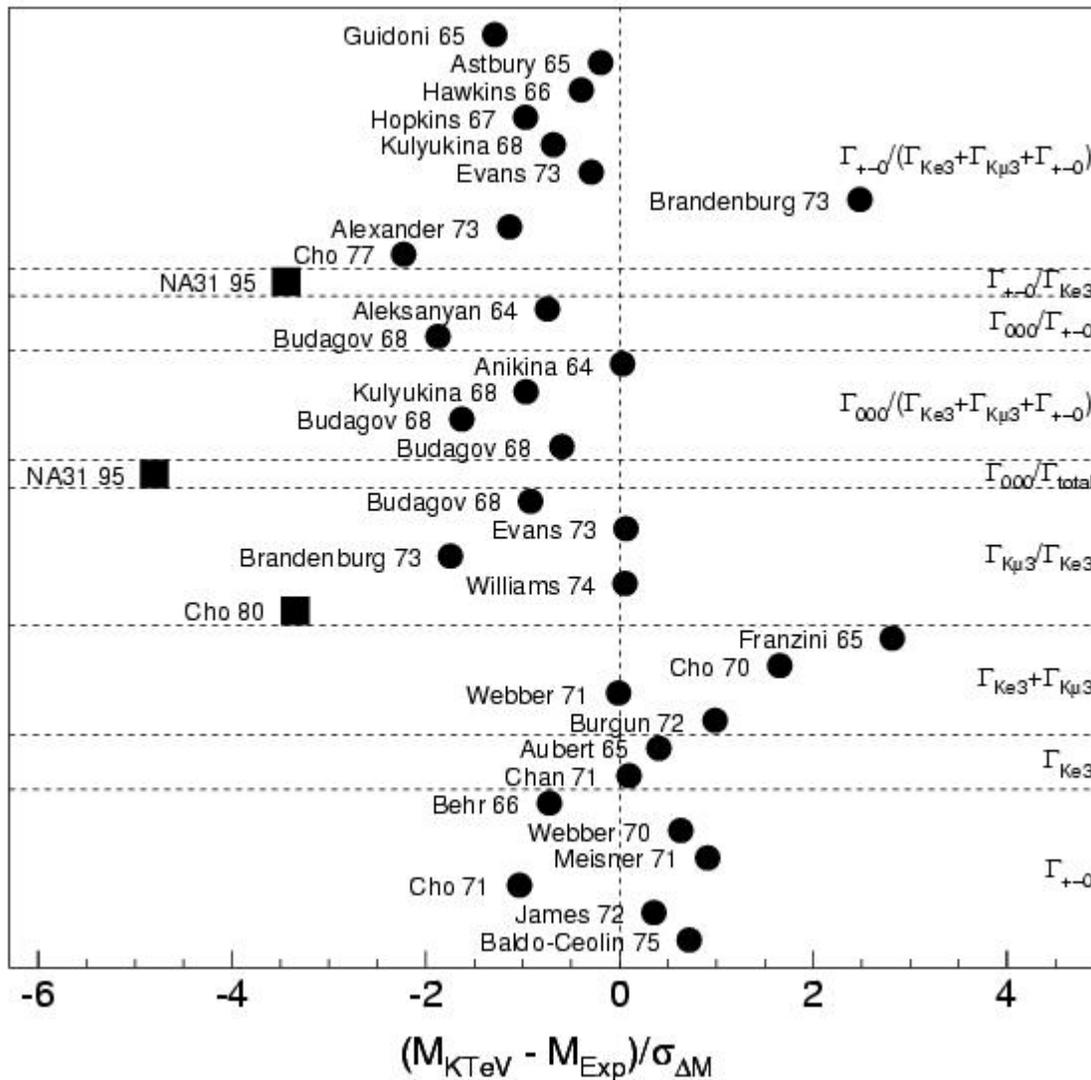
# Comparison of KTeV and PDG Branching Fractions



## Ratio of KTeV / PDG Branching Fractions

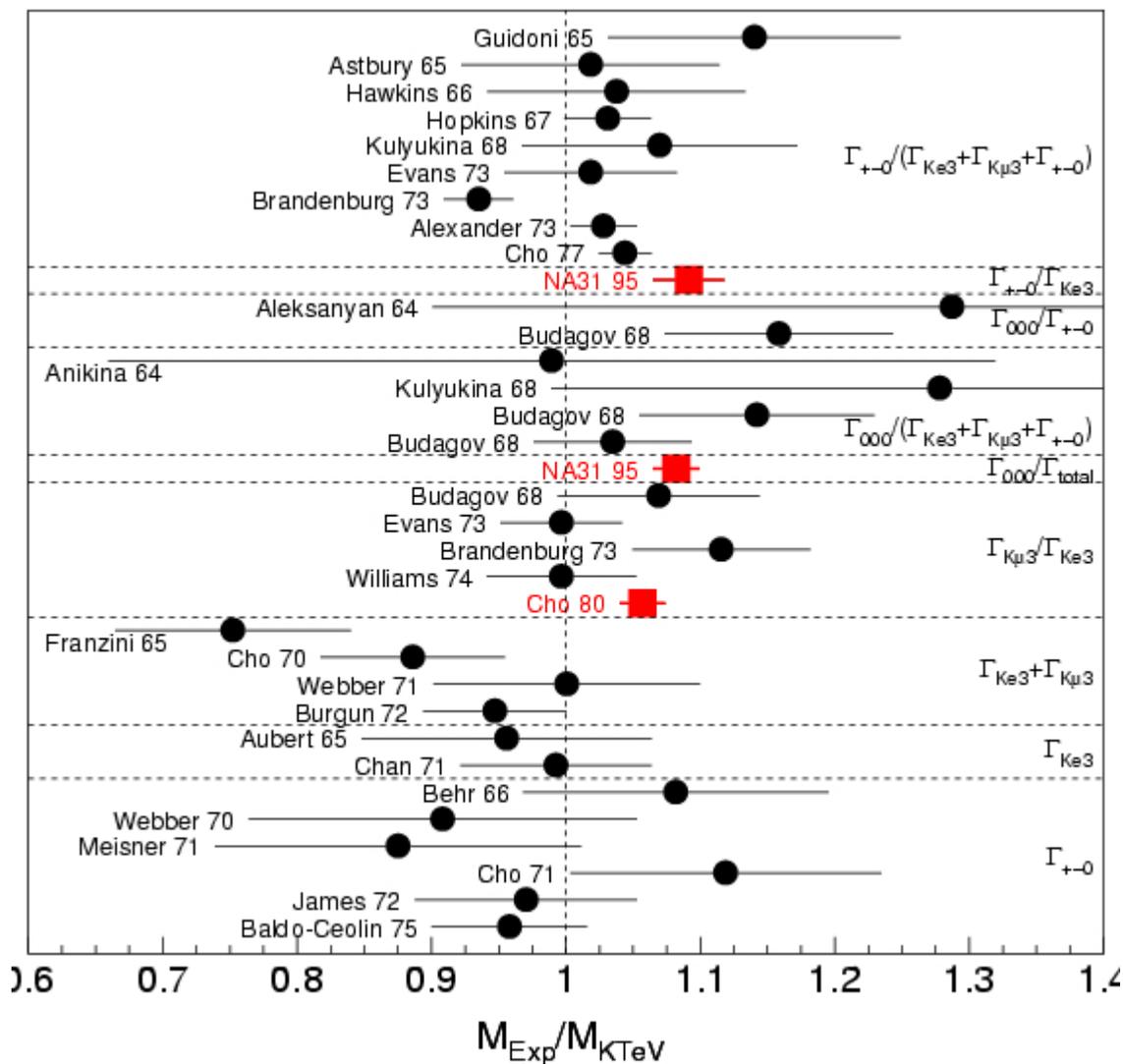


# Comparison with Individual Experiments



The dashed line at 0 represents the value based on KTeV's measured partial widths.

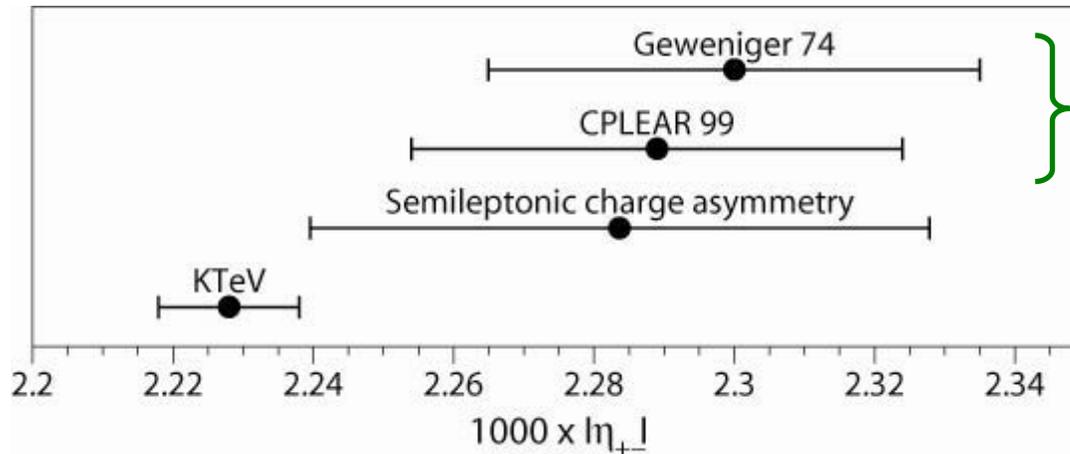
# Comparison with Individual Experiments



# Determination of $|\eta_{+-}|$ Using $B(K_L \rightarrow \pi\pi)$

$$|\eta_{+-}|^2 = \frac{\Gamma(K_L \rightarrow \pi^+ \pi^-)}{\Gamma(K_S \rightarrow \pi^+ \pi^-)} = \frac{\tau_S}{\tau_L} \frac{B_{\pi^+ \pi^-}^L + B_{\pi^0 \pi^0}^L [1 + 6 \operatorname{Re}(\varepsilon' / \varepsilon)]}{1 - B_{\pi \ell \nu}^S}$$

**KTeV:**  $|\eta_{+-}| = (2.228 \pm 0.005_{KTeV} \pm 0.009_{EXT}) \times 10^{-3}$



$K_L$ - $K_S$  Interference

## Other implications of new branching fractions:

Many rare  $K_L$  decays are normalized to decays we have remeasured.

- $K_L \rightarrow \mu^+\mu^-$  is normalized to  $K_L \rightarrow \pi^+\pi^-$  which changes by 5% compared to PDG.
- $K_L \rightarrow \gamma\gamma$  is normalized to  $K_L \rightarrow 2\pi^0$  ( $3\pi^0$ ), which changes by 8% relative to PDG.
- $\Gamma(K_L \rightarrow \mu^+\mu^-) / \Gamma(K_L \rightarrow \gamma\gamma)$  changes by  $\sim 3\%$ .

# Semileptonic Form Factor Measurements

(to determine  $I_K$  integrals)

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^\ell) C^2 |V_{us}|^2 f_+^2(0) I_K^\ell$$

$I_K$  depends on the two independent semileptonic FFs:

$f_+$  and  $f_-$  or  $f_+$  and  $f_0$

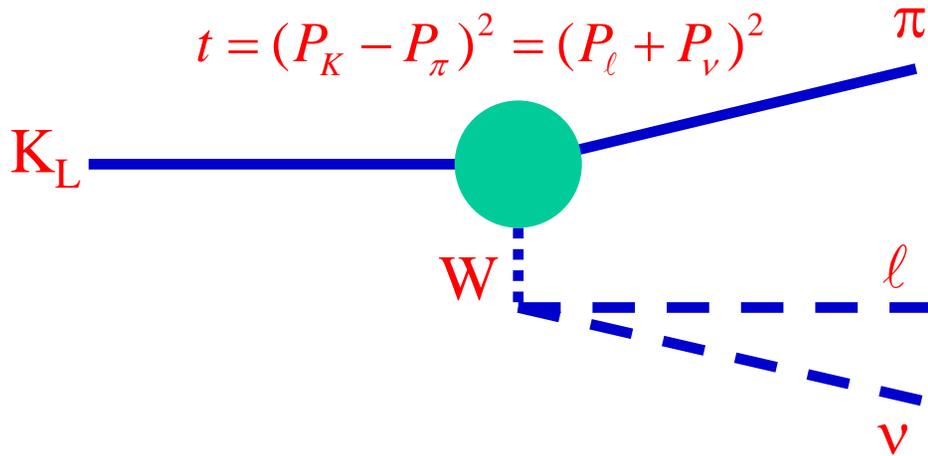
We use the following parametrization:

$$f_+(t) = f_+(0) \left( 1 + \lambda'_+ \frac{t}{M_\pi^2} + \frac{1}{2} \lambda''_+ \frac{t^2}{M_\pi^4} \right)$$

$$f_0(t) = f_+(0) \left( 1 + \lambda_0 \frac{t}{M_\pi^2} \right),$$

$$\text{where } t = (P_K - P_\pi)^2 = (P_\ell + P_\nu)^2$$

## Method to Extract Semileptonic Form Factors

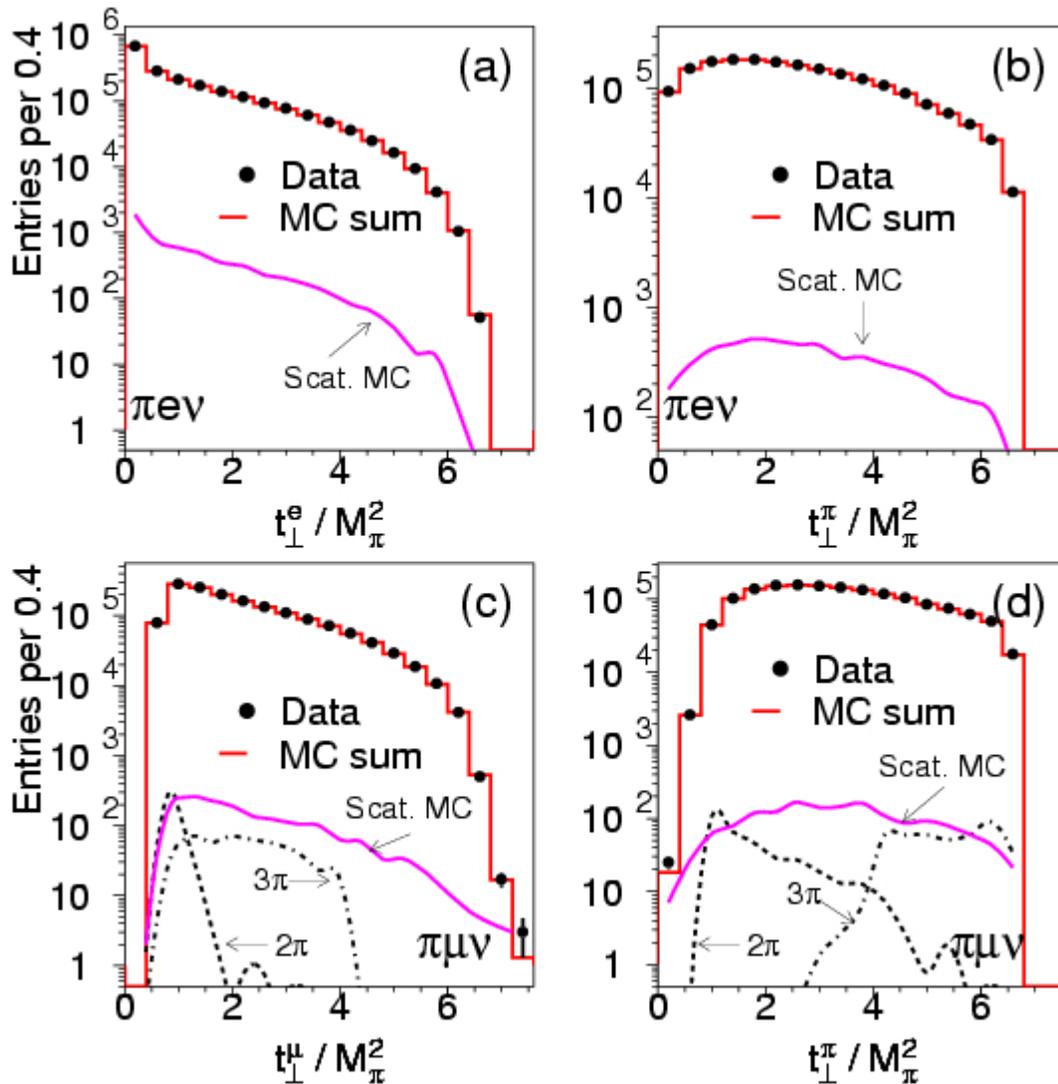


Since the kaon  $E$  is not known and the neutrino is undetected, there are two possible  $t$  values

To avoid this ambiguity, we instead study  $t_\perp$  based on components of particle momenta transverse to the kaon momentum.

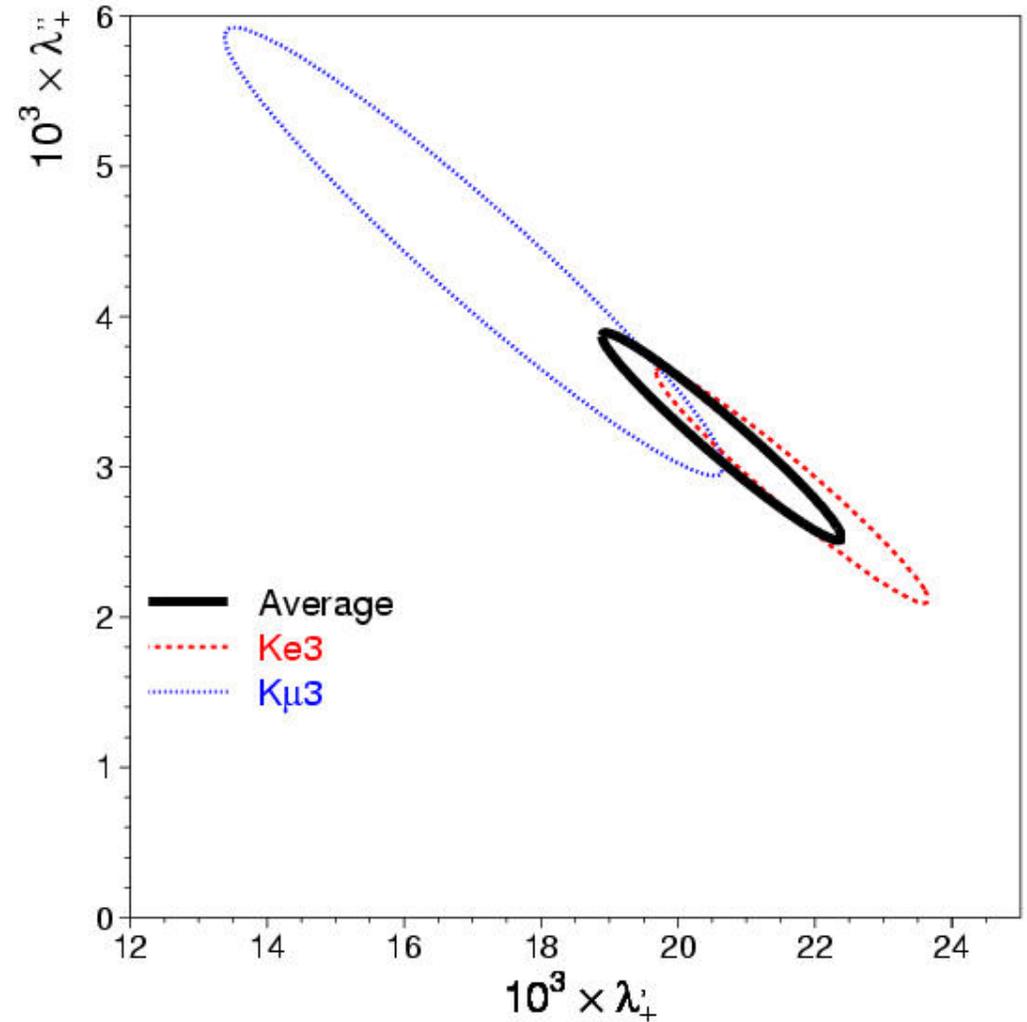
We use a Monte Carlo simulation to determine the detector acceptance as a function of  $t_\perp$  and to calculate radiative corrections.

# $t_{\perp}$ Distributions

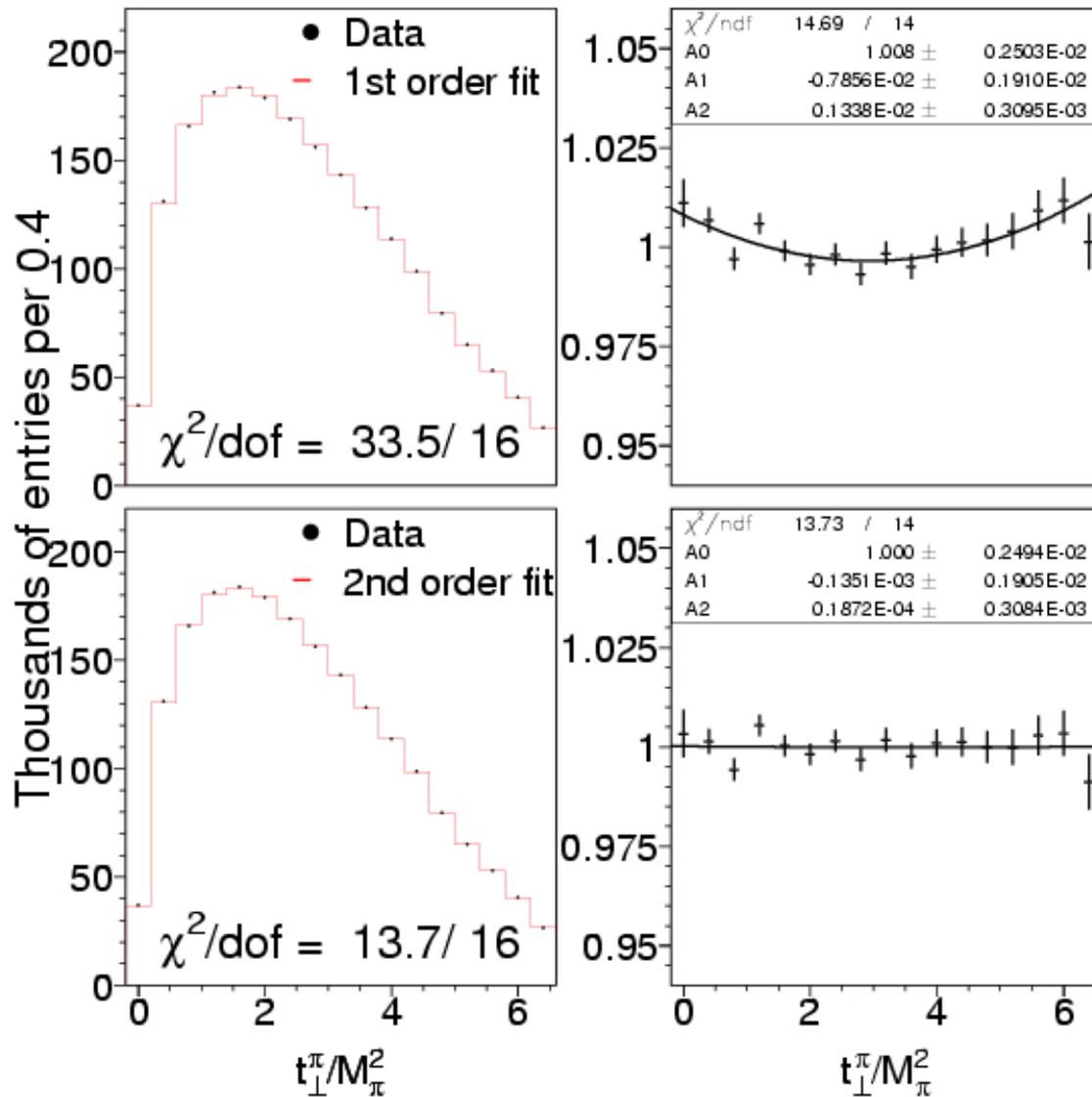


# Form Factor Results

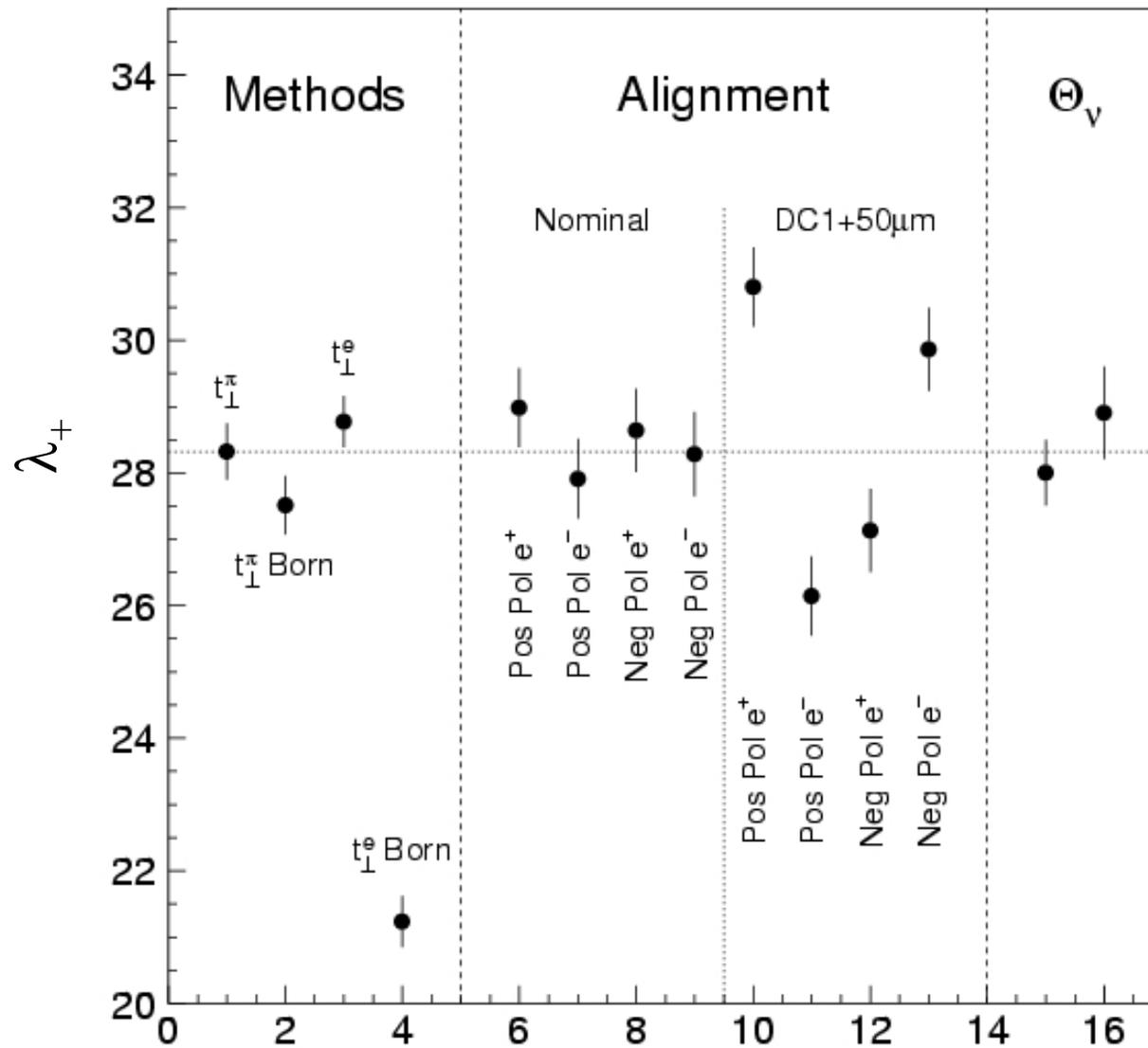
Parameter	Value ( $\times 10^{-3}$ )
$\lambda_+'$	$20.64 \pm 1.75$
$\lambda_+''$	$3.20 \pm 0.69$
$\lambda_0$	$13.72 \pm 1.31$



# First and Second Order Fits to $K \rightarrow \pi e \nu$

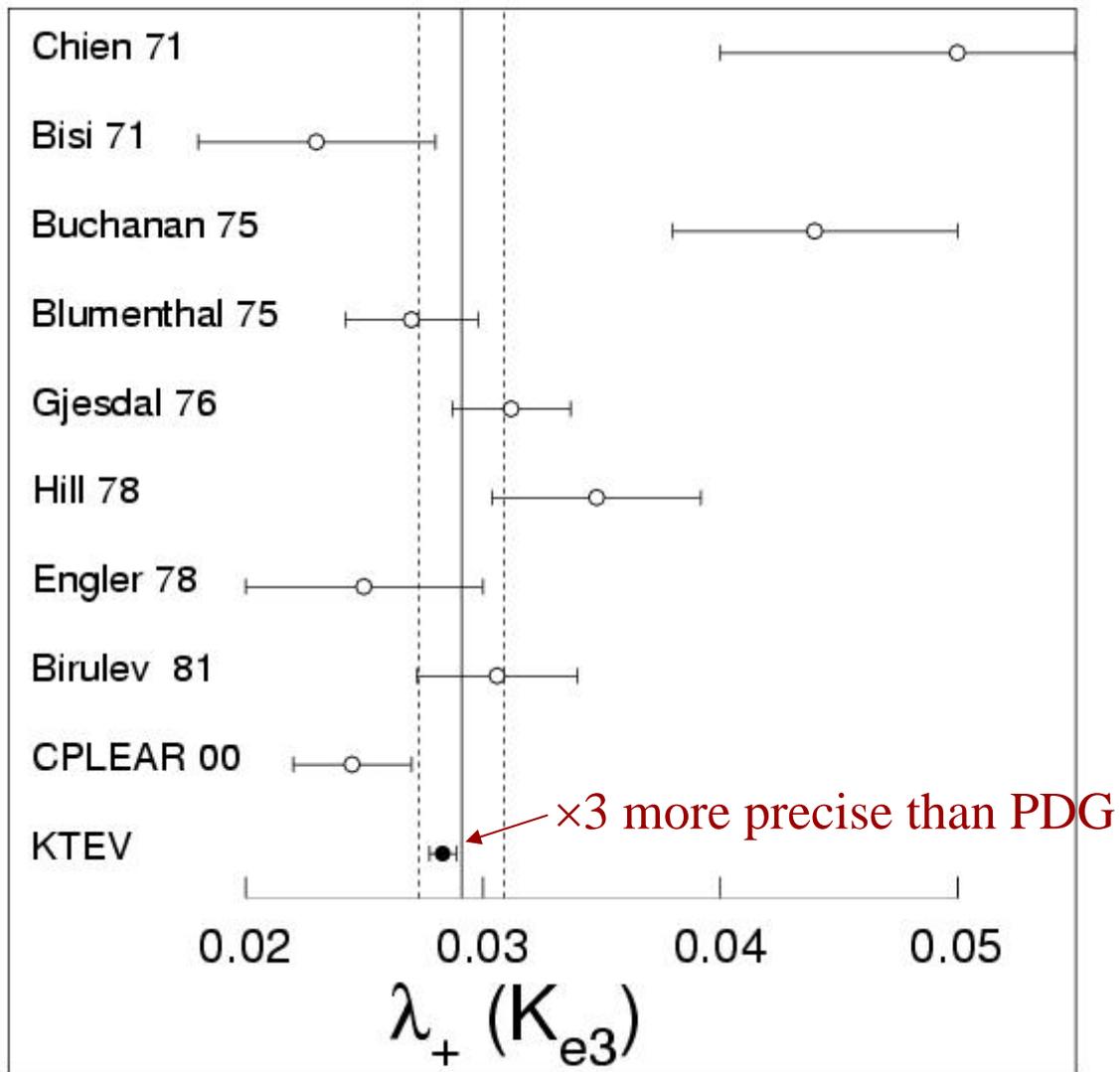


# $K_{e3}$ Form Factor Cross Checks

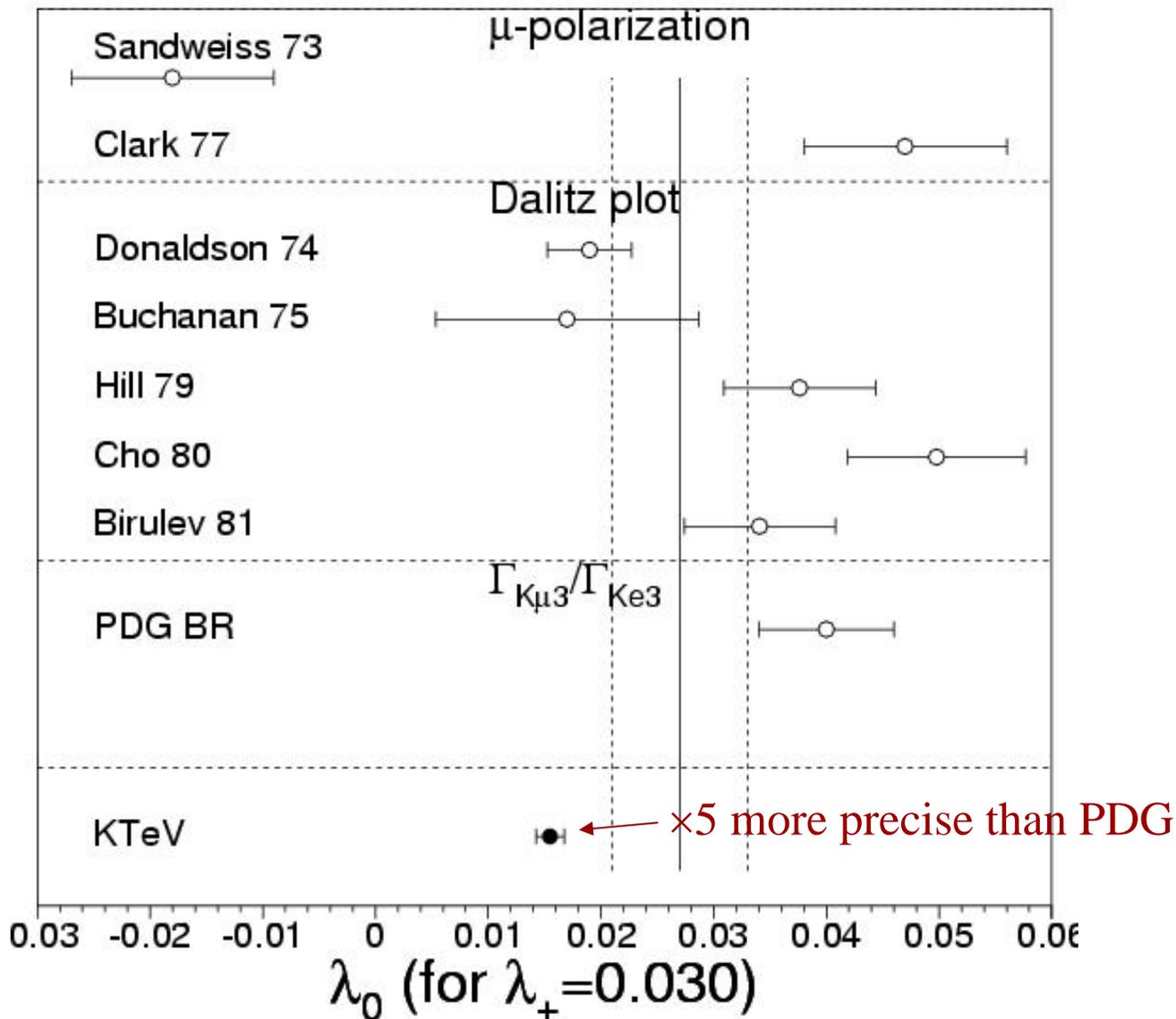


# Semileptonic Form Factors: $\lambda_+$

We use the linear parametrization to compare with previous measurements

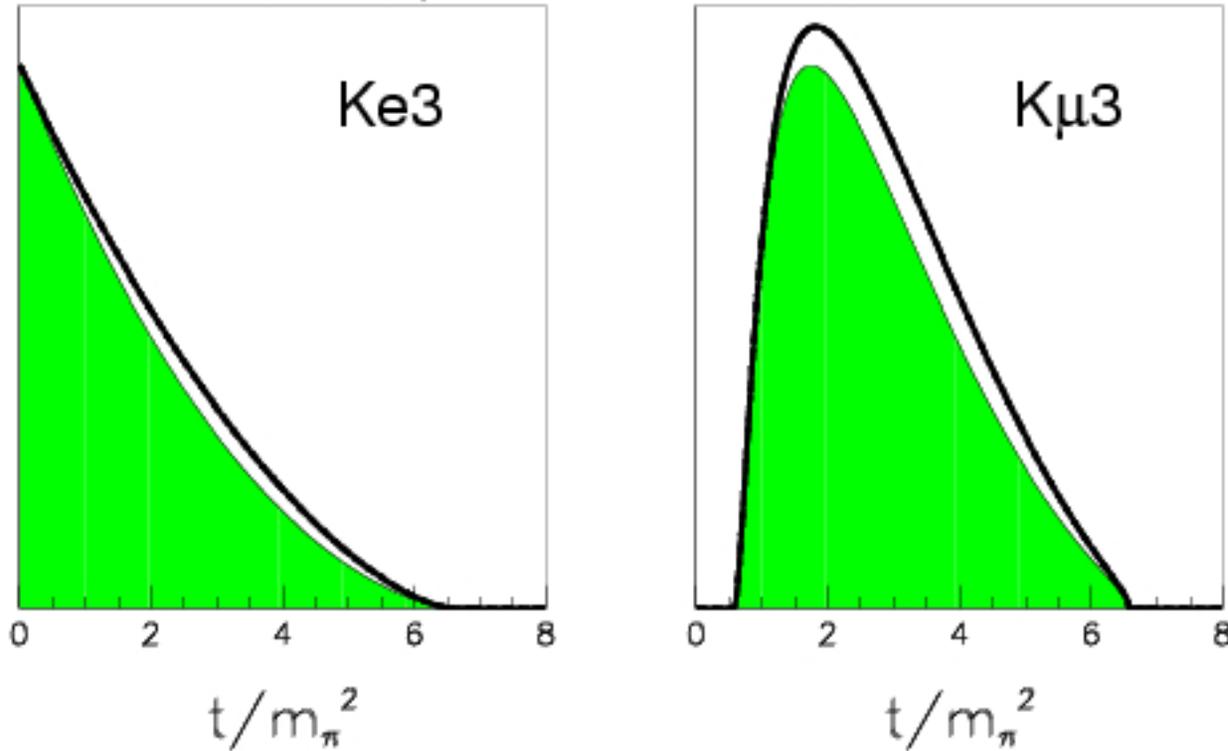


# Semileptonic Form Factors: $\lambda_0$



# Phase Space Integrals

- Phase space, no form factor
- Phase space, with form factor



$$I_K^e = 0.15350 \pm 0.00044 \pm 0.00095$$

$$I_K^\mu = 0.10165 \pm 0.00039 \pm 0.00070$$

Second error is additional 0.7% uncertainty from model dependence (pole vs. quadratic); not included in previous evaluations.

# Consistency of Branching Fraction and Form Factor Results with Lepton Universality

Compare  $\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^\ell) |V_{us}|^2 f_+^2(0) I_K^\ell$  for  $K_{e3}$  and  $K_{\mu 3}$

$$\left[ \frac{\Gamma_{K\mu 3}}{\Gamma_{Ke 3}} \right]_{PRED} = \left( \frac{1 + \delta_K^\mu}{1 + \delta_K^e} \right) \left( \frac{I_K^\mu}{I_K^e} \right)$$

1.0058(10)  
from Andre

0.6622(18) from KTeV

$$\left[ \frac{\Gamma_{K\mu 3}}{\Gamma_{Ke 3}} \right]_{MEAS} / \left[ \frac{\Gamma_{K\mu 3}}{\Gamma_{Ke 3}} \right]_{PRED} = 0.9969 \pm 0.0048 = \left( \frac{G_F^\mu}{G_F^e} \right)^2$$

Same test with PDG widths and FF gives  $1.0270 \pm 0.0182$

# Summary of $V_{us}$ Changes from KTeV Measurements

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$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^\ell) |V_{us}|^2 f_+^2(0) I_K^\ell$$

Compared to PDG:

$\Gamma_{Ke3}$  increases by 5%

$\Gamma_{K\mu 3}$  doesn't change

$I^e$  decreases by 1.7%

$I^\mu$  decreases by 4.2%

(both include -1%  
shift from  $\lambda_+$ )

## Theory Input Needed to extract $|V_{us}|$

$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^\ell) |V_{us}|^2 f_+^2(0) I_K^\ell$$

- $S_{EW}$  (short-distance rad. corr) = 1.022 (Marciano, Sirlin)
- Long-distance radiative corrections:
  - $\delta^e = 0.013 \pm 0.003$  (Andre)
  - $\delta^\mu = 0.019 \pm 0.003$  (Andre)
- $f_+(0) = 0.961 \pm 0.008$  (Leutwyler – Roos) + recent calculations

## $|V_{us}|$ Results

For  $K_L \rightarrow \pi e \nu$ :  $|V_{us}| = 0.2253 \pm 0.0023$

For  $K_L \rightarrow \pi \mu \nu$ :  $|V_{us}| = 0.2250 \pm 0.0023$

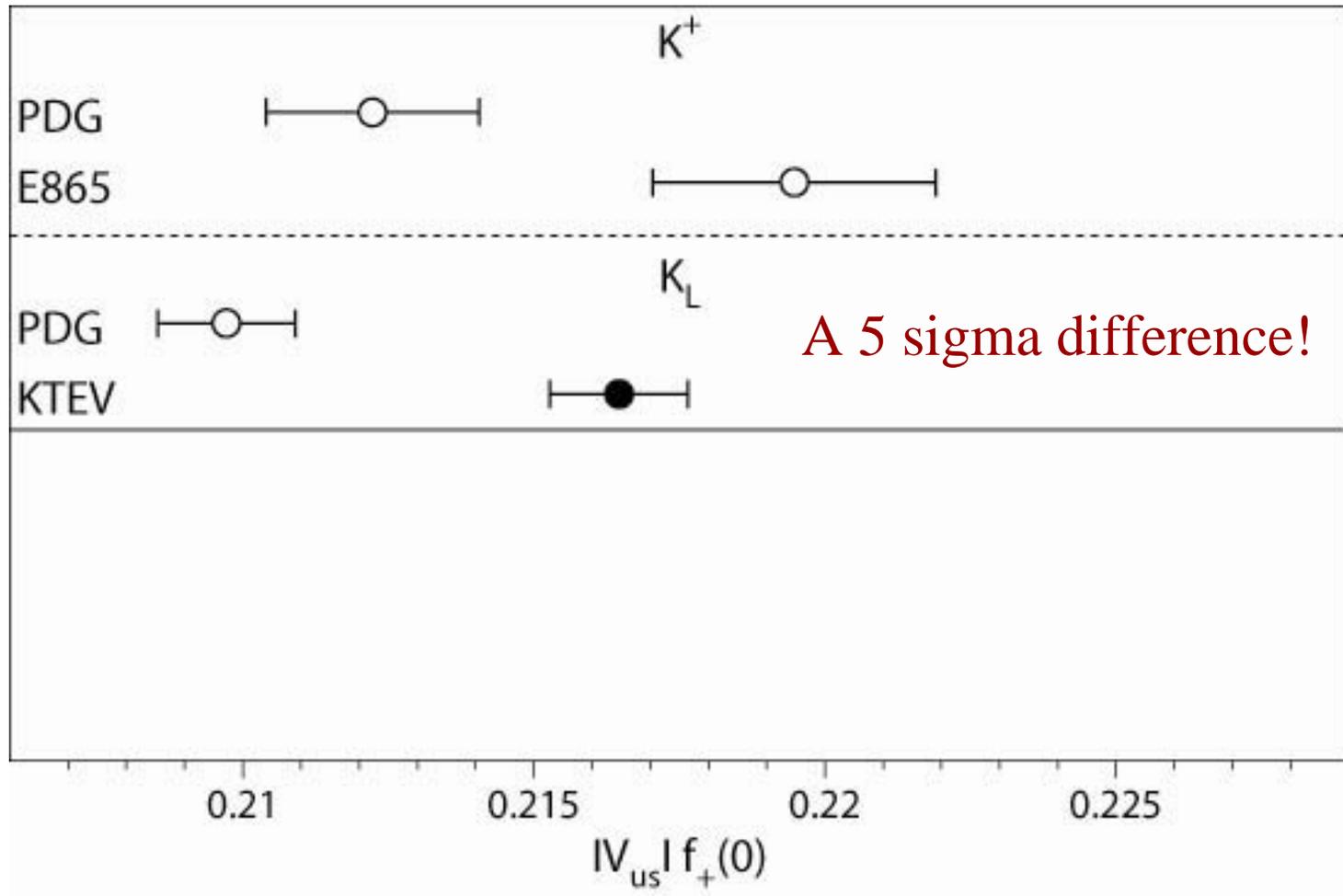
Averaging these results (accounting for correlations):

$$|V_{us}| = 0.2252 \pm 0.0008_{\text{KTeV}} \pm 0.0021_{\text{ext}}$$

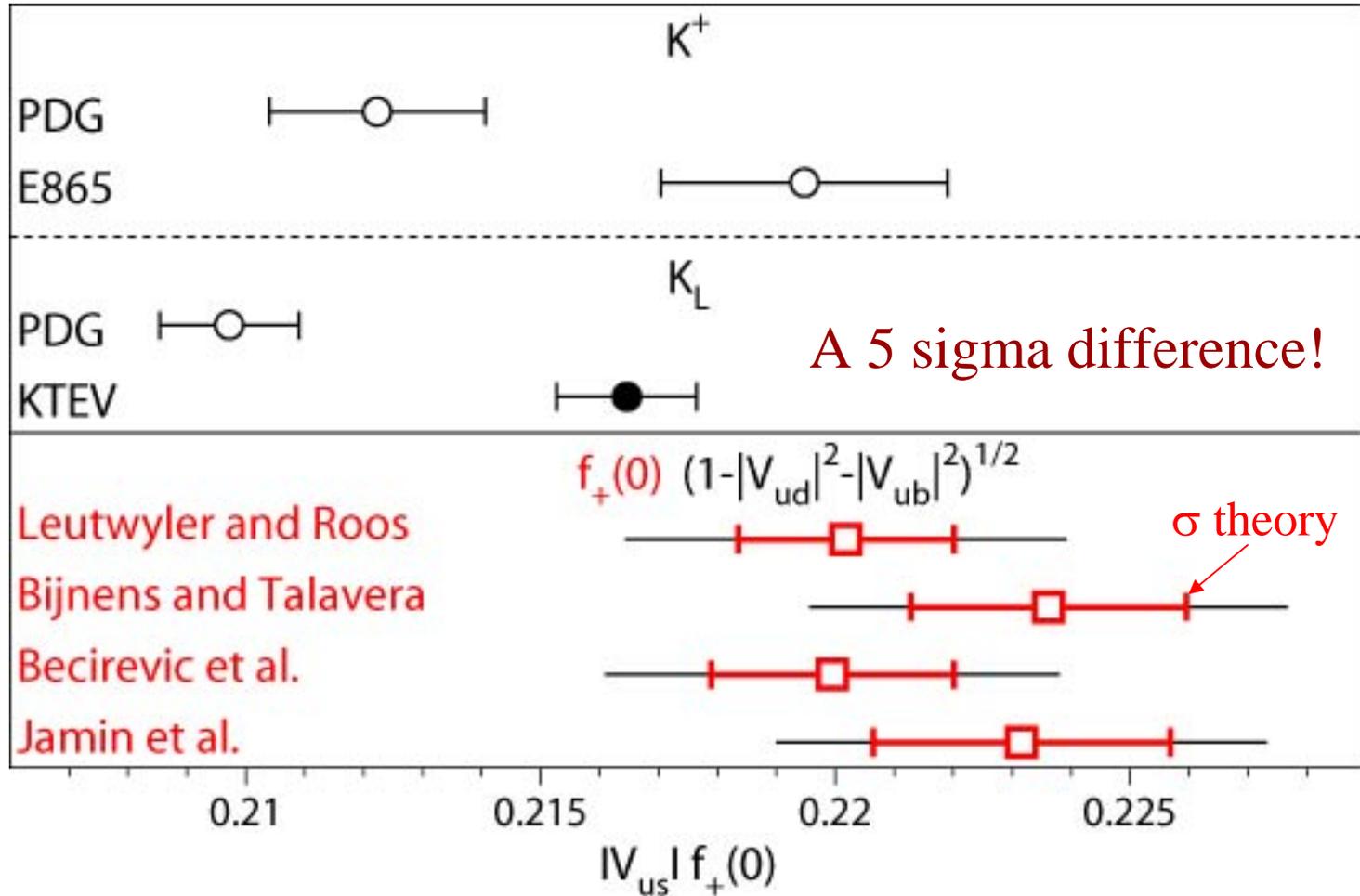
**KTeV error:** branching fractions, form factors

**Ext error:**  $f_+(0)$ ,  $K_L$  lifetime, radiative corrections

To compare with other measurements, we consider  $|V_{us}|f_+(0)$ :

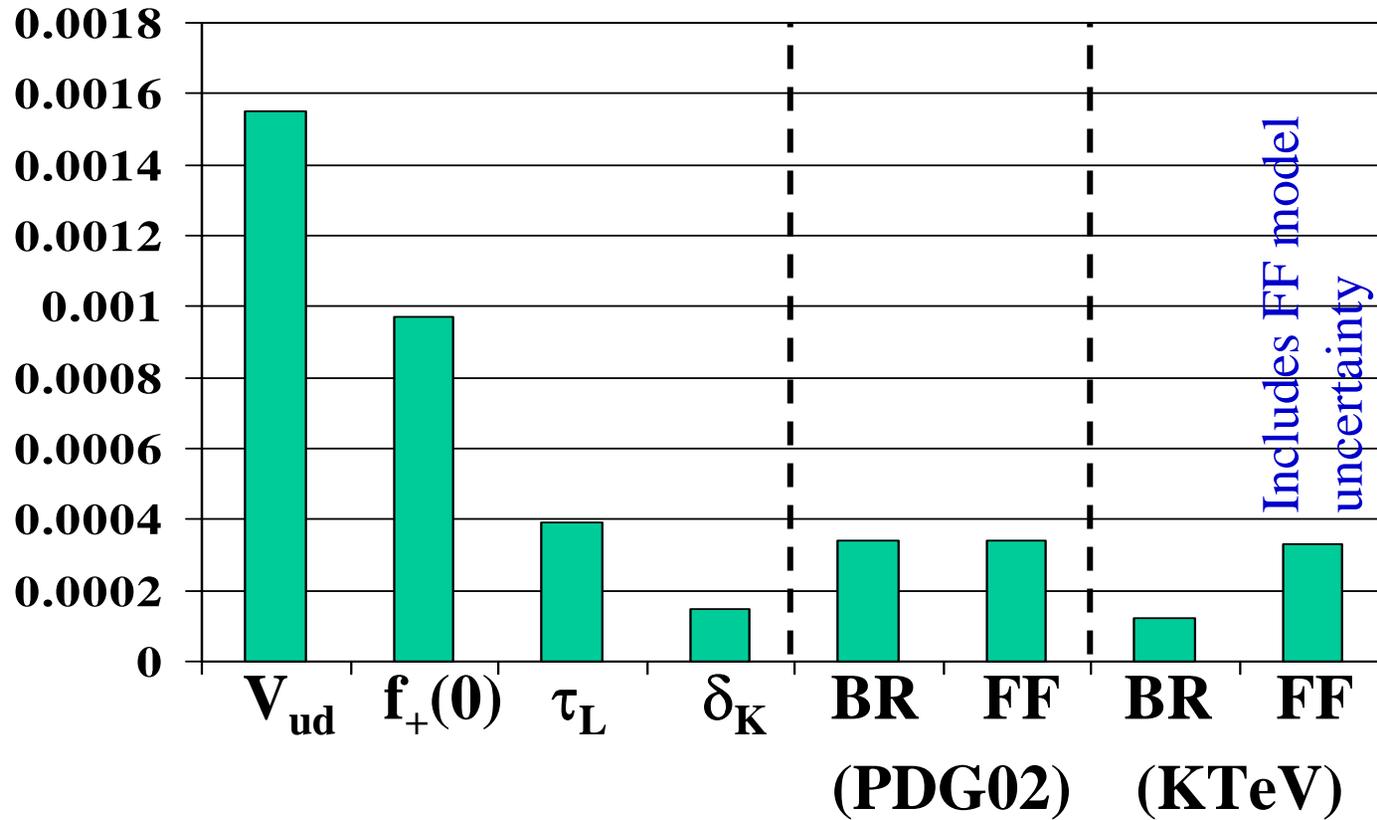


# Comparison with Unitarity

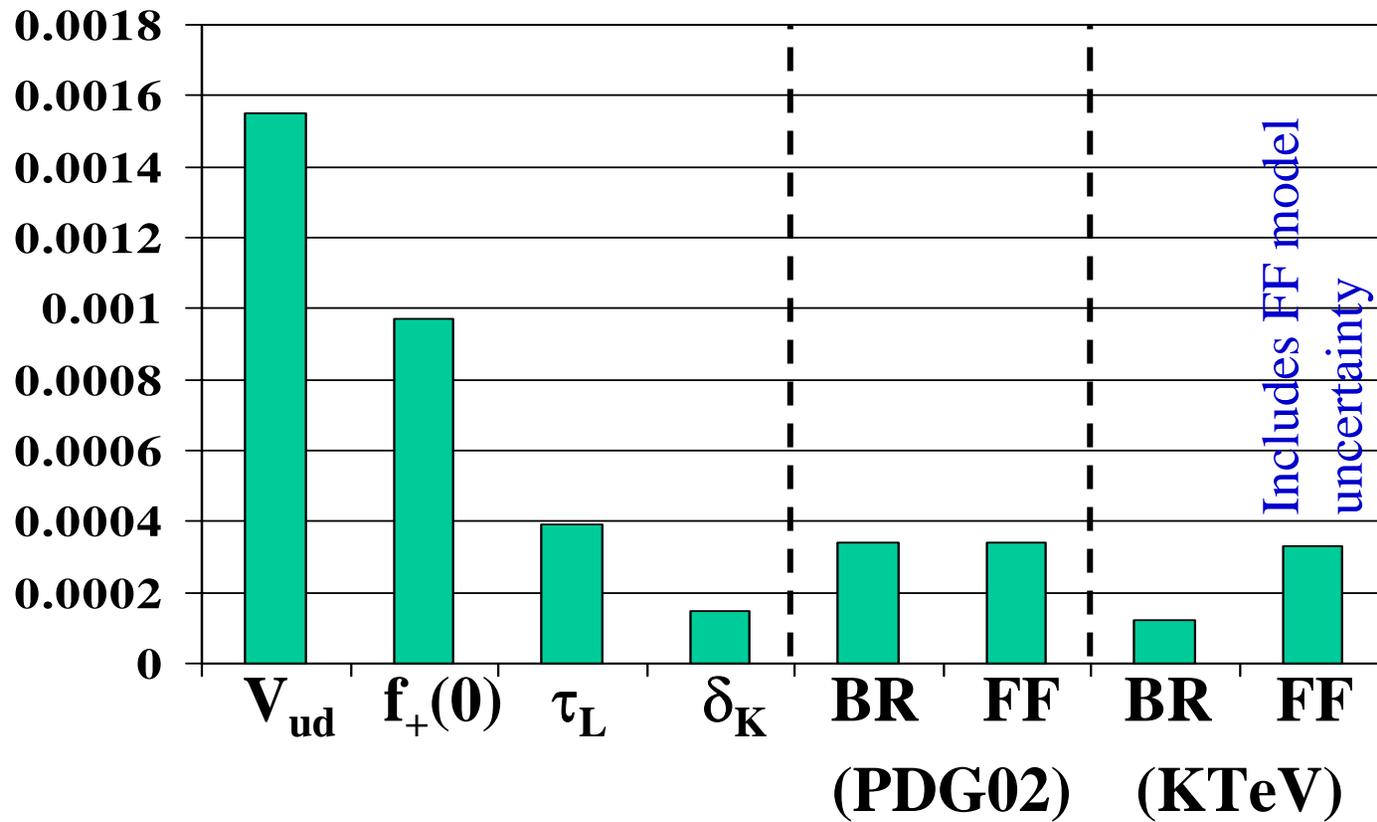


# Uncertainties on $|V_{ud}|^2 + |V_{us}|^2$

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# Uncertainties on $|V_{ud}|^2 + |V_{us}|^2$



KTeV – PDG shift in  $|V_{us}|^2$  from  $K_L$

# Conclusions

- We've made improved measurements of the six largest  $K_L$  branching fractions and semileptonic FFs; four of the six branching fractions differ from PDG by 5-8%.
- KTeV measurements result in +3% shift in  $|V_{us}|$  compared to PDG (from  $K_L$  decays); KTeV-PDG difference in is  $5\sigma$ .
- Our  $|V_{us}|$  result (based on both  $K_{e3}$  and  $K_{\mu3}$ ) is consistent with unitarity:  
$$1 - \left( |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \right) = 0.0018 \pm 0.0019$$
- KTeV  $|V_{us}|$  consistent with both PDG average and BNL E865 results using  $K^+$ .
- Our  $|\eta_{+-}|$  value is 2.6% lower than PDG.

## KTeV Publications

- T. Alexopoulos et al (KTeV), A Determination of the CKM Parameter  $|V_{us}|$ , hep-ex/0406001, submitted to Phys. Rev. Lett.
- T. Alexopoulos et al (KTeV), Measurements of  $K_L$  Branching Fractions and the CP Violation Parameter  $|\eta_{+-}|$ , hep-ex/0406002, submitted to Phys. Rev. D
- T. Alexopoulos et al (KTeV), Measurements of Semileptonic  $K_L$  Decay Form Factors, hep-ex/0406003, submitted to Phys. Rev. D.

### Supporting publication:

T. C. Andre, Radiative Corrections in  $K_{\ell 3}^0$  Decays, hep-ph/0406006, to be submitted to Phys. Rev. D.